Part I: Multiple Choices (5pts each)

(1) Classify the differential equation $e^y y'' - 5t^2 y = \sin(y)$.

(A) ordinary, linear, order 1
(B) ordinary, linear order 2
(C) ordinary, nonlinear, order 1
(D) ordinary, nonlinear, order 2
(E) partial, linear, order 1
(F) partial, linear, order 2
(G) partial, nonlinear, order 1
(H) partial, nonlinear, order 2

2. Suppose that $M(x, y)$ is the partial derivative of $F(x, y)$ with respect to $x$ and that $N(x, y)$ is the partial derivative of $F(x, y)$ with respect to $y$. Assume that all of these functions have continuous derivatives of all orders. Which of the following statements best describes the equation $\frac{dy}{dx} y(x) = -\frac{M(x, y)}{N(x, y)}$? (In the answers, $C$ is a constant.)

(A) The equation is separable.
(B) The equation is homogeneous.
(C) The equation is linear.
(D) The solution is given implicitly by $\int N(x, y) dy = -\int M(x, y) dx + C$.
(E) The solution is given explicitly as $y(x) = F(x, C)$.
(F) The solution is given implicitly by $x = F(C, y(x))$.
(G) The solution is given implicitly by $C = F(x, y(x))$.
(H) The solution has no solution.
3. The direction field picture corresponds to which of the following differential equations?

(A) $y' = 2y - 3$,  
(B) $y' = 2 + y$, 
(C) $y' = y(y + 3)$, 
(D) $y' = y(y - 3)$, 
(E) $y' = -2 - y$, 
(F) $y' = y(3 - y)$, 
(G) $y' = 2 - y$, 
(H) $y' = y - 3$.

4. Find the solution of the initial value problem $y' - 3y = 0$, $y(0) = 7$.

(A) $7e^{-3t}$  
(B) $7e^{3t}$  
(C) $e^{3t} + 7$  
(D) $e^{-3t} + 7$  
(E) $e^{\frac{1}{3}t} + 7$  
(F) $e^{\frac{1}{3}t} + 7$  
(G) $7e^{-\frac{1}{3}t}$  
(H) $7e^{\frac{1}{3}t}$

5. Find $y(1)$ if $y(t)$ is a solution of the initial value problem

\[
\frac{dy}{dt} + xy^3 = \log(1 + y), \quad y(0) = 0.
\]

(A) 0  
(B) 1  
(C) 1.57  
(D) 3.14  
(E) 2.73  
(F) 0 or 3.14  
(G) 1 or 2.73  
(H) 2 or 1.57
6. Find the solution of the given initial value problem.

\[ y' + \frac{2}{t} y = \frac{\cos(t)}{t^2}, \quad y(\pi) = 0, \quad t > 0 \]

(A) \( \sin(t) \)

(B) \( \sin(t^2) \)

(C) \( \sin(t^2) \)

(D) \( \sin^2(t) \)

(E) \( \cos(t) \)

(F) \( \cos(t^2) \)

(G) \( \cos(t^2) \)

(H) \( \cos^2(t) \)

7. Find the limit \( \lim_{t \to \infty} y(t) \) where \( y(t) \) is the solution to the initial value problem

\[ 2y' + 5y = 2, \quad y(0) = 1. \]

(A) 0

(B) 1

(C) 2

(D) 3

(E) does not exist.

8. What is the general solution to \( \frac{dy}{dx} = 2y(y + 3) \)?

(A) \( y = Ce^x \)

(B) \( y = \frac{2Ce^{3x}}{1-Ce^{3x}} \)

(C) \( y = \frac{3Ce^{x}}{1+Ce^{x}} \)

(D) \( y = \frac{2Ce^{3x}}{1+Ce^{3x}} \)

(E) \( y = \frac{3Ce^{x}}{1-Ce^{x}} \)

(F) \( y = \frac{3Ce^{3x}}{1+Ce^{3x}} \)

(G) \( y = \frac{3Ce^{6x}}{1+Ce^{6x}} \)

(H) none of the above
9. Find the solution to the initial value problem

\[ y' = \frac{3x^3 + 4x + 2}{2(y - 1)}, \quad y(-2) = 1. \]

(A) \( y = 1 + \sqrt{x^3 + 2x^2 - 2x + 4}, \) for \(-\infty < x < \infty\)

(B) \( y = 1 - \sqrt{x^3 + 2x^2 - 2x + 4}, \) for \(-\infty < x < \infty\)

(C) \( y = 1 + \sqrt{x^3 - 2x^2 + 2x + 4}, \) for \(-2 \leq x \)

(D) \( y = 1 - \sqrt{x^3 - 2x^2 + 2x + 4}, \) for \(-2 \leq x \)

(E) \( y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}, \) for \(-\infty < x < \infty\)

(F) \( y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}, \) for \(-2 \leq x,\)

(G) none of the above

10. At time \( t = 0, \) a tank contains 50 lb of salt dissolved in 100 gal of water. Assume that the water containing \( \frac{1}{4} \) lb of salt/gal is entering the tank at a rate of 3 gal/min and that the well-stirred mixture is draining from the tank at the same rate. Find the limiting amount \( Q_\infty \) salt that is present after a very long time.

(A) 100 gal

(B) 50 gal

(C) 25 gal

(D) 0.25 gal

(E) 3 gal

(F) 15 gal

(G) none of the above

11. At any time \( t, \) the rate of growth of the population \( N \) of deer in a state park is proportional to the product of \( N \) and \( L - N, \) where \( l = 500 \) is the maximum number of deer the park can maintain. When \( t = 0, N = 100 \) and \( t = 4, N = 200. \) Find the amount of deer when \( t = 1? \)

(A) 100

(B) 121

(C) 121.06

(D) 200

(E) 200.10

(F) 500

(G) none of the above
12. How many solutions does the equation \( y' = \frac{\cos(t)y^2}{t+y^2} \) have satisfying the initial value \( y(0) = 1 \)?

(A) 0
(B) 1
(C) 2
(D) 3
(E) infinite

13. How many solutions does the initial value problem have?
\[ y' = y^{1/3}, \quad y(0) = 0. \]

(A) 0
(B) 1
(C) 2
(D) 3
(E) infinite

14. Suppose a certain population satisfies the initial value problem
\[ \frac{dP}{dt} = 50P(6000 - P), \quad P(0) = 2000. \]
Choose the statement that correctly describe the concavity of the solution curve.

(A) The graph is concave up everywhere.
(B) The graph is concave down everywhere.
(C) There is an inflection point where \( P = 2000 \).
(D) There is an inflection point where \( P = 3000 \).
(E) There is an inflection point where \( P = 3500 \).
(F) None of the above.

15. Which one of the following equations is exact?

(A) \((2x + 4y) + (2x - 2y)y' = 0.\)
(B) \((x \ln(y) + xy)dx + (y \ln(x) + xy)dy = 0.\)
(C) \( \frac{dy}{dx} = -\frac{ax + by}{tx + cy} \).
(D) \( \frac{dy}{dx} = -\frac{ax - by}{tx - cy} \).
16. Consider the initial value problem \( y' = 3 + x - y, \ y(0) = 1 \). Use Euler's method, with \( h = 0.1 \) to approximate the value \( y(0.3) \).

(A) 0.935  
(B) 1.013  
(C) 1.125  
(D) 1.235  
(E) 1.347  
(F) 1.432  
(G) 1.571  
(H) 1.613  
(I) 1.674

17. Solve the differential equation

\[(y \cos(x) + 2xe^y) + (\sin(x) + x^2e^y - 1)y' = 0.\]

(A) \( y \sin(x) + x^2e^y = C \).  
(B) \(-y \sin(x) + xe^y - y = C \).  
(C) \(6 \cos(x) + x^2e^y - y = C \).  
(D) \(y \sin(x) + x^2e^y - y = C \).  
(E) None of the above.

18. Let \( f(x, y), \frac{\partial f}{\partial y}(x, y) \) be continuous functions on a rectangular region \( R \) containing the point \((t_0, y_0)\). Suppose that \( f(x, y) \neq 0 \) for any \((x, y) \in R\). How many solutions does the following initial value problem have near the point \((t_0, y_0)\)?

\[ \frac{dy}{dx} = \frac{1}{f(x, y)}, \ y(t_0) = y_0. \]

(A) no solution  
(B) at least 1, but may not be unique  
(C) unique one solution  
(D) none of the above
19. (10pts) A home buyer can afford to spend no more than 800 dollar per month on mortgage payments. Suppose that the interest rate is 9%, and the term of the mortgage is 20 years. Assume that interest is compounded continuously and the payments also made continuously.

(1) Determine the maximum amount that this buyer can afford to borrow.
(2) Determine the total interest paid during the term of the mortgage.