EXAM I
Math 109 / Music 109A, Spring 2004

Name ____________________________ Id ____________________

Each problem is worth 10 points.
(1) **Aural**: Notate the rhythm (one measure each).

(a) \[ \frac{4}{4} \]  
(b) \[ \frac{12}{8} \]

Circle the interval between the two notes.

(c) minor third  
   major third  
   fifth  
   octave  

(d) minor third  
   major third  
   fifth  
   octave

Circle the triad type.

(e) minor  
   major  
   diminished  
   augmented

(2) For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm \( n = qm + r \):

(a) 3, 77 ;

(b) 11, \(-55k + 16\), where \( k \) some integer.

(3) Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches). Use the space on page 4 if you need it.

(a) \( f(x) = -(x - 1)^2 \)  
(b) \( g(x) = 1 + \sin 2x \)
(4) Write the indicated note as a whole note on the given staff, choosing an appropriate clef.

(a) \( B_4^b \) \hspace{1cm} (b) \( G_2 \) \hspace{1cm} (c) \( A_3^4 \)

(5) Name the type of chord given by each of the following sequences of intervals. The intervals are given in semitones.

(a) \( (3, 4, 5) \) \hspace{1cm} (b) \( (4, 4, 4) \)

(c) \( (3, 3, 3, 3) \) \hspace{1cm} (d) \( (4, 3, 3, 2) \)

(6) For the set \( \{(a, b) \in \mathbb{Z}^2 \mid b \neq 0\} \) show that the relation \( \sim \) defined by \( (a, b) \sim (a', b') \) iff \( ab' - a'b = 0 \) is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers \( \mathbb{Q} \).

OR

For the set \( \mathbb{Z} \) and a fixed positive integer \( m \), show that the relation \( \equiv \) defined by \( k \equiv \ell \) iff \( m \mid k - \ell \) is an equivalence relation. Explain why there are exactly \( m \) equivalence classes.
(7) For the following modes and tonic notes, indicate the appropriate key signature on
the given staff: (Parts (a)-(c) are 2 points apiece; part (d) is 4 points.)

(a) Locrian with tonic F²

(b) Dorian with tonic B

(c) Lydian with tonic E♭

(d) Add the needed sharps or flats to notes so that the following gives the Phrygian
scale tones 1 to 8, from C to C.

(8) Extend the following melody with two measures having the same rhythm, employing
these transpositions:
   (a) diatonic down one scale tone in the second measure
   (b) chromatic up a minor third (from the original) in the third measure

(9) Give the duration in beats of:
   (a) a doubly-dotted eighth note in $\frac{2}{4}$ time.
   (b) a quarter note in $\frac{6}{8}$ time (compound time signature).
   (c) an eighth note 7-tuplet in $\frac{4}{4}$ time.
(10) Give the interval ratio of:

(a) an octave

(b) down a fourth

(c) a minor sixth

Given that A₄ is tuned to 440 Hz, give the frequencies of:

(d) G⁴₄

(e) B⁵₃