EXAM I
Math 109 / Music 109A, Spring 2005

Name ___________________________ Id _______________________

Each problem is worth 10 points.

(1) For the following pairs of integers \(m, n\), find the numbers \(q\) and \(r\) whose existence is asserted in the division algorithm \(n = qm + r\):

(a) 5, 78 ;

(c) 7, \(-21k + 13\), where \(k\) some integer.

(2) Sketch the graph of the function \(g(x) = -1 + \cos 2x\) by starting with a more basic function and applying one or more geometric transformations (shifts or stretches).
(3) For the set \( \{(a, b) \in \mathbb{Z}^2 \mid b \neq 0\} \) show that the relation \( \sim \) defined by \( (a, b) \sim (a', b') \) iff \( ab' - a'b = 0 \) is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers \( \mathbb{Q} \).

OR

For the set \( \mathbb{Z} \) and a fixed positive integer \( m \), show that the relation \( \equiv \) defined by \( k \equiv \ell \) iff \( m \mid k - \ell \) is an equivalence relation. Explain why there are exactly \( m \) equivalence classes.

(4) Write the indicated note as a whole note on the given staff, choosing an appropriate clef.

\[
\begin{align*}
\text{(a)} & \quad \text{\rule{3cm}{0.5mm}} \\
\text{B}_2^\flat & \\
\text{(b)} & \quad \text{\rule{3cm}{0.5mm}} \\
\text{A}_5^\sharp &
\end{align*}
\]

Identify these keyboard intervals:

\[
\begin{align*}
\text{(c)} & \quad \text{\rule{1.5cm}{0.5mm}} \\
\text{(d)} & \quad \text{\rule{1.5cm}{0.5mm}}
\end{align*}
\]
(5) For the following modes and tonic notes, indicate the appropriate key signature on the given staff:

(a) Phrygian with tonic B

(b) Locrian with tonic E

(6) Transpose this melodic excerpt, written in C minor, up to F² minor. Preserve the scale-tone spelling of each melody note.

(7) Give the duration in beats of:

(a) a half note in $\frac{6}{8}$ time (compound time signature).

(b) a dotted eighth note in $\frac{2}{2}$ time.

(c) an eighth note 5-tuplet in $\frac{4}{4}$ time.

(8) On the line below notate and name the following tuplets:

(a) that which divides the half note into 3 equal notes

(b) that which divides the quarter note into 5 equal notes

(c) that which divides the whole note into 11 equal notes
(9) Complete these measures with a single durational note:

(a) \( \begin{array}{c}
\text{\( \frac{4}{4} \)}
\end{array} \)

(b) \( \begin{array}{c}
\text{\( \frac{5}{4} \)}
\end{array} \)

(c) \( \begin{array}{c}
\text{\( \frac{12}{4} \)}
\end{array} \)

(10) For the song *Mary Had A Little Lamb*, give the form (e.g., AABC) by dividing it into segments consisting of two bars. Locate and identify a translation other than that which comes from the overall form.

Mary had a little lamb, little lamb,

Little lamb, Mary had a little lamb, his

Fleece was white as snow