EXAM I
Math 109 / Music 109A, Spring 2003

Name ___________________________________________ Id _________________________

(1) Aural: Circle the interval between the two notes.

(a) minor third (b) minor third (c) minor third
   major third major third major third
   fifth      fifth      fifth
   octave    octave    octave

Notate the rhythm (one measure each).

(d) \( \frac{4}{4} \)   (e) \( \frac{12}{8} \)

(2) For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm \( n = qm + r \):

(a) 5, 78 ;
(b) 3, −295 ;
(c) 7, −21k + 16, where \( k \) some integer.

(3) Sketch the graphs of these functions by starting with a more basic function and applying one or more geometric transformations (shifts or stretches).

(a) \( f(x) = -x^2 \)   (b) \( g(x) = -1 + \cos 2x \)
(4) For the set \( \{(a, b) \in \mathbb{Z}^2 \mid b \neq 0\} \) show that the relation \( \sim \) defined by \((a, b) \sim (a', b') \) iff \( ab' - a'b = 0 \) is an equivalence relation. Explain how the set of equivalence classes are in one-to-one correspondence with the set of rational numbers \( \mathbb{Q} \).

**OR**

For the set \( \mathbb{Z} \) and a fixed positive integer \( m \), show that the relation \( \equiv \) defined by \( k \equiv \ell \) iff \( m \mid k - \ell \) is an equivalence relation. Explain why there are exactly \( m \) equivalence classes.

(5) Write the indicated note as a whole note on the given staff, choosing an appropriate clef.

(a) \( G_4 \)  
(b) \( B_2^b \)  
(c) \( A_5^\# \)

(6) Identify these keyboard intervals:

(a) \( \text{BZ} \)  
(b) \( \text{BE} \)  
(c) \( \text{A9} \)
(7) For the following modes and tonic notes, indicate the appropriate key signature on
the given staff:

(a) Phrygian with tonic B

(b) Locrian with tonic G♯

(c) Lydian with tonic B♭

(8) Add the needed sharps or flats to notes so that the following gives the Dorian scale
tones 1 to 8, from C to C.

(9) Transpose this melodic excerpt, written in C minor, up to E minor. Preserve the
scale-tone spelling of each melody note.

(10) Give the duration in beats of:

(a) a half note in $\frac{6}{8}$ time (compound time signature).

(b) a dotted eighth note in $\frac{2}{2}$ time.

(c) an eighth note 5-tuplet in $\frac{4}{4}$ time.