EXAM II
Math 109 / Music 109A, Spring 2004

Name ___________________________ Id ___________________

(1) Express each of these intervals as elements of \( \mathbb{R}^+ \) three ways: (1) as a radical or the reciprocal of a radical, (2) as a power of 2, and (3) by a decimal approximation with 2 digits to the right of the decimal.

(a) up 43 cents

(c) up a major third

(d) the interval from \( E^3_5 \) to \( F_4 \)

(2) A string on a stringed instrument has length 20 in. Indicate the positions of three frets which will allow the string to play the sequence of notes which goes up by major (keyboard) thirds ending on the octave.

(3) Evaluate these logarithms without a calculator. Write down each step of the simplification.

(a) \( \log_5 \left( \frac{\sqrt[3]{25}}{5} \right) \)

(b) \( \log_{\sqrt{7}} 49 \)

(c) \( \log_a \left( \frac{1}{\sqrt[4]{a^3}} \right) \)
(4) On the same axes, sketch the graphs of \( f_1(x) = \log_2 x \) and \( f_2(x) = \log_4 x \), plotting a few key points. Identify and quantify any horizontal or vertical stretches or shifts that relate the two graphs. Justify your answer.

(5) Convert the interval ratio \( r \) to the additive measurement indicated. Round off to 2 digits to the right of the decimal.

(a) \( r = \frac{5}{4} \) to cents

(b) \( r = 3 \) to semitones

(c) \( r = \frac{7}{6} \) to octaves

(6) Determine the base \( b \) for which the function \( f(x) = \log_b x \) converts interval ratios to 11-chromatic units. (This means an octave should correspond to a distance of 11 on the logarithmic axis.)

(7) In a given mode, the tonic triad is the chord consisting of scale tones \( \hat{1}, \hat{3}, \text{and} \hat{5} \). For the following modes, classify the tonic triad as major, minor, diminished, or augmented.

(a) Dorian

(b) Lydian

(c) Locrian

(d) Phrygian
(8) Identify each chord in this major mode passage by root note with suffix (e.g., E♭⁷). Write your labeling under each chord.

(9) Write these chords in root position in the given mode and key signature with correct spelling:

(a) IIIaug, minor mode

(b) Im⁷, Phrygian mode

(c) bVI⁷, major mode

(10) Complete the following to a four-part harmonization of the given melody (key of C) using only whole notes, so that the melody is the top part and the bottom note is always the root. The final chord should be the tonic triad I (as labeled below), and the two intermediate chords should be seventh chords which follow the circle of fifths to the final I. Label the chords in measures 2 and 3 by letter above the chord and by scale note below the chord, with appropriate suffix.