EXAM III
Math 109 / Music 109A, Spring 2003

Name _______________________________ Id ______________________

(1) **Aural**: Identify the harmonic by circling the correct integer.

(a) 2 3 4 5 6 7
(b) 2 3 4 5 6 7
(c) 2 3 4 5 6 7
(d) 2 3 4 5 6 7

(2) For each given fundamental (1st harmonic), identify by letter and subscript the note which is the keyboards best approximation of its $k^{th}$ harmonic. Calculate in cents how sharp or flat the keyboard note is compared to the harmonic.

(a) G$_3$, $k = 3$

(b) A$_5^2$, $k = 7$

(c) C$_1$, $k = 11$

(3) Using laws of exponents, prove that $\log_b(xy) = \log_b x + \log_b y$. 

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(4) Sketch the graphs of (a) \( f(x) = 2^x \) and (b) \( g(x) = 3^x \) on the same set of coordinate axes. Indicate whether they differ by a horizontal or vertical stretch/compression, and identify the stretch factor which converts (a) to (b).

(5) Express the interval ratio \( \frac{3}{2} \) (the “just fifth”) in terms of \( n \)-chromatic units, for the given values \( n \). Round off to 2 digits to the right of the decimal.

(a) \( n = 7 \)

(b) \( n = 12 \)

(c) \( n = 13 \)

(d) \( n = 19 \)

(6) Define group homomorphism and show that, for \( b > 0, b \neq 1 \), the function \( g(x) = \log_b x \) defines a group homomorphism from \((\mathbb{R}^+, \cdot)\) to \((\mathbb{R}, +)\).
(7) What is the value of $\phi(8)$ ($\phi$ being the Euler phi function)? Identifying the group of modular 8-chromatic intervals with $\mathbb{Z}_8$, list all the generating intervals for the 8-chromatic scale. Pair each generating interval with its additive inverse, and for each such pair $[k],[\ell]$, draw the circle of intervals which proceeds by intervals of $[k]$ in one direction, $[\ell]$ the other.

(8) Create the 5-tone row chart for the original row: [0] [3] [2] [4] [1]. Express each entry as an element of $\mathbb{Z}_5$ in the form $[r]$, where $0 \leq r < 5$.

(9) Classify each of these sets with given associative law of composition as monoid, group, both, or neither. Justify your answer.

(a) $(\mathbb{Z}, +)$

(b) $(\mathbb{Q} - \{0\}, \cdot)$

(c) $(\mathbb{Z}^+, \cdot)$
(10) Complete the following to a four-part harmonization of the given melody (key of C) using only whole notes, so that the melody is the top part and the bottom note is always the root. The final chord should be the tonic triad I (as labeled below), and the two intermediate chords should be seventh chords which follow the circle of fifths to the final I. Label the chords in measures 2 and 3 by letter above the chord and by scale note below the chord, with appropriate suffix.