Homework 1
Math 109 / Music 109A, Spring 2004

Due Monday, February 1.

(1) For the following pairs of integers $m,n$, find the numbers $q$ and $r$ whose existence is asserted in the division algorithm:

(a) $11, 101$ ;

(b) $10, -7, 276, 854$ ;

(c) $19, 19^7 + 5$ ;

(d) $10, 35b + 43$, where $b$ an even integer.

(2) Sketch the graphs of these functions, and indicate how each is a geometric transformation (shift or stretch) of a simpler function:

(a) $f(x) = x + 1$

(b) $f(x) = (x - 3)^2$

(c) $f(x) = \cos(\frac{x}{2})$

(3) For each of the following sets and relations determine whether or not an equivalence relation has been defined, explaining which of the three defining conditions are satisfied and which are not.

(a) The set of people alive now; “was born the same year as”.

(b) $\mathbb{R}; >,$

(c) $\mathbb{Z};$ for a fixed positive integer $n$, $\equiv$ defined by $k \equiv \ell$ iff $n \mid k - \ell$.

(d) The set of keyboard note classes; $\sim$ defined by $N \sim N'$ iff the interval between $N$ and $N'$ is either the unison interval or a major third (up or down).

(4) For the set \{$(a, b) \in \mathbb{Z}^2 | b \neq 0$\} show that the relation $\sim$ defined by $(a, b) \sim (a', b')$ iff $ab' - a'b = 0$ is an equivalence relation and that the set of equivalence classes is in one-to-one correspondence with $\mathbb{Q}$. 
(5) Identify these keyboard notes by letter and subscript (e.g., D₃ or A₇⁴):

(a) 

(b) 

(c) 

(d) 

(6) Identify these keyboard intervals:

(a) 

(b) 

(c) 

(d) 

(7) Write on staff paper (choosing an appropriate clef), and name with subscript, the note which is:

(a) a fourth below E₄.

(b) a minor sixth above C₅⁴.

(c) a minor ninth below G₆⁴.

(d) a tritone above B₇⁴.

(8) Consider an arbitrary seven-note scale whose sequence of (seven) intervals contains only the numbers 1 and ½. (The standard scale is one example.) Is it possible that such a scale has a non-trivial cyclic permutation which is equivalent to itself? If so, give an example; if not, explain why not. (Hint: How many appearances of 1 and of ½ must there be?)

(9) For the following modes and tonic notes, indicate the appropriate key signature on staff paper:

(a) Phrygian with tonic C⁴.

(b) Dorian with tonic C.

(c) Locrian with tonic B⁴.

(d) Aolian with tonic F⁴.

(10) Transpose this melodic excerpt, from Theme From Exodus, written in D minor, up to F minor. Preserve the scale-tone spelling of each melody note.