Homework 1

Due Monday, January 29.

(1) For the following pairs of integers \( m, n \), find the numbers \( q \) and \( r \) whose existence is asserted in the division algorithm:

(a) 15, 49 ;

(b) 8, -37 ;

(c) 3, 3^{32} + 7 ;

(d) 5, 25k + 9, where \( k \) is some integer.

(2) Sketch the graphs of these functions, and indicate how each is obtained by geometric transformations (shifts and/or stretches) of simpler functions:

(a) \( f(x) = 2x + 1 \)

(b) \( f(x) = \frac{1}{2}x^2 - 1 \)

(c) \( f(x) = -1 + \sin(3x) \)

(3) For each of the following sets and relations determine whether or not an equivalence relation has been defined. Explain why or why not. If so, describe the set of equivalence classes.

(a) The set of people having a single residence in the United States; “lives in the same state as as”.

(b) \( \mathbb{R} ; \leq \).

(c) \( \mathbb{Z} ; \) for a fixed positive integer \( n, \equiv \) defined by \( k \equiv \ell \) iff \( n \mid k - \ell \).

(d) The set of keyboard note classes; \( \sim \) defined by \( N \sim N' \) iff the interval between \( N \) and \( N' \) is either the unison interval or a major third (up or down).

(4) For the set \( \{(a, b) \in \mathbb{Z}^2 \mid b \neq 0\} \) show that the relation \( \sim \) defined by \( (a, b) \sim (a', b') \) if and only if \( ab' - a'b = 0 \) is an equivalence relation and that the set of equivalence classes is in one-to-one correspondence with \( \mathbb{Q} \).
(5) Identify these notes by letter and subscript (e.g., D₃ or A₄): 

(a) \( \begin{align*} &D3 \\ &D_{\flat} \end{align*} \) (b) \( \begin{align*} &C\#3 \\ &C_{\#} \end{align*} \) (c) \( \begin{align*} &E\#3 \\ &E_{\#} \end{align*} \) (d) \( \begin{align*} &B3 \\ &B_{\flat} \end{align*} \)

(6) Identify these intervals by name and number of semitones:

(a) \( \begin{align*} &D3-D3 \\ &D_{\flat}-D_{\flat} \end{align*} \) (b) \( \begin{align*} &C\#3-C\#3 \\ &C_{\#}-C_{\#} \end{align*} \) (c) \( \begin{align*} &E\#3-E\#3 \\ &E_{\#}-E_{\#} \end{align*} \) (d) \( \begin{align*} &B3-B3 \\ &B_{\flat}-B_{\flat} \end{align*} \)

(7) Write on staff paper, and name with subscript, the note which is:

(a) a major third above D₃.
(b) a fourth above F₃.
(c) a major ninth below G₆.
(d) a tritone above E₂.

(8) Notate all the key signatures on a line of staff paper. Order the keys using flats by ascending number of flats and similarly for those using sharps. For each indicate the major key and the minor key it denotes.

(9) For the following modes and tonic notes, indicate the appropriate key signature on staff paper:

(a) Lydian with tonic C.
(b) Dorian with tonic F.
(c) Locrian with tonic G₆.
(d) Phrygian with tonic G.

(10) Transpose this melodic excerpt, written in C minor, up to E minor. Preserve the scale-tone spelling of each melody note.