Homework 4
Math 109 / Music 109A, Spring 2004

Due Monday, March 22.

(1) Evaluate without a calculator by writing the argument of log as a power of the base. Write down each step of the simplification, e.g., \( \log_3 3^{\sqrt{3}} = \log_3 3^{\frac{3}{2}} = \frac{3}{2} \log_3 3 = \frac{3}{2} \):

(a) \( \log_{10}(0.01) \)  \hspace{1cm} (b) \( \log_2 16 \)  \hspace{1cm} (c) \( \log_5 \sqrt[3]{5} \)  \hspace{1cm} (d) \( \log_c \sqrt[4]{c} \)

Express as a single logarithm without coefficient, i.e., in the form \( \log_b c \) (do not evaluate with a calculator):

(e) \( \log_2 5 + \log_2 3 \)  \hspace{1cm} (f) \( \log_4 7 - 2\log_4 11 \)

(g) \( \log_3 10 + \log_3 16 \)  \hspace{1cm} (h) \( 2\log_a x^2 - \frac{1}{2} \log_{\sqrt{a}} x \)

(2) Sketch the graphs of:

(a) \( f(x) = 5^x \)  \hspace{1cm} (b) \( g(x) = \log_5 x \)  \hspace{1cm} (c) \( r(x) = 3^x \)  \hspace{1cm} (d) \( s(x) = \log_3 x \)

Determine which pairs of these functions are inverse to each other, and which pairs differ by a horizontal or vertical stretch/compression. In the latter case, identify the stretch factor, justifying your answer.

(3) (a) Using laws of exponents, prove these properties of logarithms:

\[ \log_b \frac{x}{y} = \log_b x - \log_b y \]
\[ \log_b (x^p) = p \log_b x \]

(b) Suppose \( n \in \mathbb{Z}^+ \) and we want the interval of an octave to correspond to a distance of \( n \) on a logarithmic axis parameterizing pitch. What base should we choose? Justify your answer.

(4) Convert to semitones the intervals given by the following ratios: (Round off to 2 digits to the right of the decimal.)

(a) 3  \hspace{1cm} (b) 0.8  \hspace{1cm} (c) \( \frac{4}{3} \)  \hspace{1cm} (d) \( \sqrt{2} \)  \hspace{1cm} (e) \( e \)

Convert to cents the intervals given by the following ratios, rounding off to the nearest whole cent:

(f) 1.25  \hspace{1cm} (g) 1.1  \hspace{1cm} (h) \( \frac{7}{4} \)  \hspace{1cm} (i) \( \frac{2}{3} \)  \hspace{1cm} (j) \( \pi \)
(5) Write on the staff the note which best approximates the frequency having the given interval ratio \( r \) from the given note:

\[
\begin{align*}
\text{(a)} & & \text{(b)} & & \text{(c)} & & \text{(d)} \\
\text{\text{\flat F}} & & \text{\# G} & & \text{\natural F} & & \text{\natural G} \\
\text{D} & & \text{C} & & \text{D} & & \text{F} \\
\text{r = 3} & & \text{r = } \frac{2}{5} & & \text{r = 2.3} & & \text{r = } \pi^{-1}
\end{align*}
\]

(6) Compose a melody accompanied by a harmonization which exhibits the following features (as before):

(a) Some symmetry in its overall form (e.g. ABAC or ABA)

(b) Some usage of one or more of these transformations:
   (i) translation (melodic or rhythmic)
   (ii) transposition (diatonic or chromatic)
   (iii) retrogression

(c) Some symmetry in its overall form (e.g. ABAC or ABA)

The harmonization should be written on different staffs from that of the melody. It may be in the form of sustained chords or arpeggio. This composition may be a modification of the one created for Homework 2, or it may be a totally new work. This should be submitted electronically, as in Homework 2, as an AAC file. Please name the file by your last name and first initial, followed by HW4.m4a, as in wrightdHW4.m4a.

You should submit with your written assignment a hard copy of the musical score (You may print out the Finale file.) which analyzes the harmony by note class with suffix above the staffs, and by Roman numeral with suffix below the staffs. Scores turned in without the sound file will not receive credit.