Homework 4
Math 109 / Music 109A, Spring 2005

Due Monday, March 21.

(1) Express each of these intervals as elements of $\mathbb{R}^+$ three ways: (1) as a radical or the reciprocal of a radical, (2) as a power of 2, and (3) by a decimal approximation with 3 decimal digits.
   
   (a) up 67 cents
   (b) down 1050 cents
   (c) up a major sixth
   (d) the interval from $B_3$ to $G_1^4$

(2) Assuming $A_4$ is tuned to 440 Hz, find the frequencies for:
   
   (a) $C_4$  (b) $D_2^4$  (c) $F_3$  (d) $E_1^5$

   Suppose middle C is tuned as 256 Hz. Find the frequencies for:
   
   (a) $A_4$  (b) $G_6^6$  (c) $C_1$  (d) $F_2^5$

(3) For each of these chords, voiced within an octave with the root on the bottom, give the pitch of each note in the chord. Assume $A_4$ is tuned to 440 Hz.

   (a) major triad with root $E_3^4$
   (b) minor triad with root $F_4^4$
   (c) minor seventh chord with root $A_5^4$
   (d) diminished seventh with root $A_3^5$

(4) Determine whether each pair of musical intervals, expressed as elements of $\mathbb{R}^+$, are equivalent modulo octave. Explain why or why not.

   (a) 5, 20  (b) 14, $\frac{7}{2}$  (c) 2.3, 9.2  (d) 1.04, 0.13  (e) $\pi$, $\frac{3\pi}{2}$

(5) Suppose a string on a banjo has length 40cm. Indicate positions of the 12 frets which will allow the string to play one octave of the ascending chromatic scale.

(6) Prove that if $y = f(t)$ has period $P$, then so does $y = f(t) + c$, $y = f(t - c)$, and $y = cf(t)$, for any $c \in \mathbb{R}$. Prove that $f(t/c)$ ($c \neq 0$) has period $cP$.
(7) Suppose the function \( y = f(t) \) is the periodic function of period \( P \) corresponding to a musical tone, and suppose the graph of \( y = f(t) \) is:

For each of the functions below, sketch its graph and explain how its associated tone compares that of \( f(t) \).

(a) \( y = \frac{1}{2} f(t) \)  
(b) \( y = f(2t) \)  
(c) \( y = \tilde{f}(t) + c \)  
(d) \( y = f(t + c) \)

(8) Find the value \( \alpha \) for which the pitch associated to the periodic function \( y = \sin(\alpha t) \), where \( t \) is time in seconds, is:

(a) middle C  
(b) \( A_5^2 \)  
(c) \( D_4^6 \)

(9) Find the period, frequency, amplitude, and phase shift for these functions, and express each in the form \( A \sin(\alpha t) + B \cos(\alpha t) \):

(a) \( f(t) = 5 \sin(30\pi t + \frac{\pi}{4}) \)  
(b) \( g(t) = \sqrt{2} \sin(800t + \pi) \)  
(c) \( h(t) = -\frac{5}{3} \sin(2000t + \arcsin(0.7)) \)

(10) Find the period, frequency, amplitude, and phase shift for these functions, and express each in the form \( d \sin(\alpha t + \beta) \):

(a) \( f(t) = 4 \sin(300t) + 5 \cos(300t) \)  
(b) \( g(t) = 2 \sin(450\pi t) - 2 \cos(450\pi t) \)  
(c) \( h(t) = -\sin(1500\pi t) + 3 \cos(1500\pi t) \)