Problem Set 4

Hand in all.

(1) Find all normal subgroups of $\mathfrak{S}_5$ and $\mathfrak{A}_5$. [Hint: you want to use (II.I.5)(iv) here. See the examples that follow it.]

(2) Let H be a proper subgroup of a finite group G. Prove that $G \neq \bigcup_{g \in G} \imath_g(H)$.

(3) Read Theorem 1.12 (and its proof) in Jacobson and use it to do exercise 13 on p. 79 of Jacobson.

(4) [Jacobson p. 58 #6] Let G_1 and G_2 be simple groups. Show that every normal subgroup of $G = G_1 \times G_2$ other that G and $\{1\}$ is isomorphic to either G_1 or G_2.

(5) If $HK = G$ for $H, K \leq G$, then $|H||K| = |G||H \cap K|$. [See Jacobson p. 58 #9.]

(6) Compute the automorphism groups $\text{Aut}(G)$, $\text{Inn}(G)$, and $\text{Out}(G)$ for $G = D_4$. [Hint: try labeling automorphisms by where they send r and h.]

(7) Construct explicitly a homomorphism Φ from the quaternions Q to the Klein 4-group V_4 with kernel $\{\pm 1\}$.