Problem Set 6

(Hand in all.)

(1) Determine all subgroups of the mod 3 Heisenberg group (II.M.17). For the normal ones, determine the quotient group.

(2) Let $\theta\colon H \to \mathrm{Aut}(K)$ be a group homomorphism. Prove that $K \rtimes H$ (one usually writes this instead of $\rtimes_\theta$) sits in a *split* short exact sequence with K on the left and H on the right.

(3) Classify the groups of order 490 up to isomorphism. Start by proving that none of them are simple. [Edit: this problem is a bit too hard I think. More reasonable is this: classify the groups of order 245 up to isomorphism. Now, what can you say *about* the classification of groups of order 490? You don't have to do a complete classification, but say something about it and attempt a partial classification.]

(4) Do Jacobson p. 79 #10. (by a famous theorem, these wreath products $G \wr H$ actually turn out to contain all extensions of H by G as subgroups. You may also want to look at #16 on p. 84, but you don't have to hand it in.

(5) Up to rotational symmetry, how many different ways can you paint the edges of a tetrahedron red, green, or blue?

(6) Give a short proof (without Burnside) of the result that a finite group G acting transitively on a finite set X (with at least 2 elements) has at least one $g \in G$ that acts without fixed points. [Hint: consider the union of the stabilizers G_x, and don't forget that transitivity means that X is one big orbit.]