

SCHWARZIAN DERIVATIVES AND UNIVALENCE OF HARMONIC MAPPINGS

PETER DUREN

University of Michigan

In collaboration with Martin Chuaqui and Brad Osgood, the speaker recently extended the classical notion of Schwarzian derivative of an analytic function to general complex-valued harmonic functions $f(z) = h(z) + \overline{g(z)}$, where h and g are analytic. With respect to the conformal parameter $\lambda = |h'| + |g'|$ of the associated minimal surface, the definition is

$$\mathcal{S}f = 2\{(\log \lambda)_{zz} - (\log \lambda)_z^2\}.$$

Nehari's well-known general criterion (1954) for univalence of an analytic function in the unit disk is now generalized to the Weierstrass–Enneper lift of a harmonic mapping f to its minimal surface. The criterion involves $\mathcal{S}f$ and the Gauss curvature of the surface, which can also be computed in terms of λ . (Joint work with M. Chuaqui and B. Osgood)