THE GROWTH OF UNIVALENT FUNCTIONS WITH AN INITIAL GAP

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Abstract

We consider the class S_p of those functions

$$f(z) = z + a_{p+1}z^{p+1} + \dots$$

univalent in the unit disk Δ. We show that if $f \in S_p$, where p is large, then

$$\alpha(f) = \lim_{n\to\infty} \frac{|a_n|}{n} \leq \alpha(p) = \frac{2(\log p \, \log\log p)^2}{p^4}.$$

We also obtain upper bounds for $|a_n|$ when $f \in S_p$.

In the opposite direction, examples are constructed of functions $f \in S_p$ for which $\alpha(f) \geq C_0 p^{-4}$, where C_0 is a positive absolute constant.