

Title: Complex interpolation of compact operators - an update.

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Abstract: In 1960 Mark Krasnoselskii showed that the Riesz-Thorin interpolation theorem has the following “compactness” analogue:

If T is a bounded linear map of L^p to itself for $p = p_0$ and for $p = p_1$ and, furthermore, $T : L^{p_0} \rightarrow L^{p_0}$ is also compact, then $T : L^p \rightarrow L^p$ is compact for all p between p_0 and p_1 .

There are, or seem to be, natural extensions of Krasnoselskii’s result, in particular, in the context of Alberto Calderón’s marvellous theory of complex interpolation spaces $[A_0, A_1]_\theta$. In particular one may ask this:

Suppose that $T : A_0 \rightarrow A_0$ is compact and $T : A_1 \rightarrow A_1$ is bounded. Is $T : [A_0, A_1]_\theta \rightarrow [A_0, A_1]_\theta$ compact for all θ between 0 and 1?

In 1963, Calderón showed that the answer to this question is yes, provided that A_0 and A_1 satisfy certain extra conditions. Today, forty-four years after Calderón’s work, we know many more special cases where the answer is yes. But we still do not know the answer in general to this question.

In this talk I will survey various facets of this problem, and mention related matters, including the beautiful Rochberg-Semmes description of Calderón’s spaces as “points” along a “geodesic”, and an application of known compactness results to P.D.E.s.

Further remarks: To see more about this problem in general, you are welcome to visit the web address

<http://www.math.technion.ac.il/~mcwikel/compact>

More specifically, to see a list of various topics that I may or may not discuss in this talk, as the audience prefers and as time permits, you can look at the rather more detailed and more eccentric abstract of a related talk which I gave in Uppsala earlier this month. It is at

<http://www.math.uu.se/inform/abstracts/cwikel.pdf>

The topics which I plan to emphasize this time, are those numbered there as 2, 8, 3, 4 and 5.