

Small Deformations of Banach Algebras
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ABSTRACT. A deformation of a normed algebra (\mathcal{A}, \cdot) is a new multiplication \times defined on the same space \mathcal{A} such that

$$\|a \cdot b - a \times b\| \leq \delta \|a\| \|b\|, \quad \text{for } a, b \in \mathcal{A},$$

for some small constant δ . A linear functional F on an algebra (\mathcal{A}, \cdot) is almost multiplicative if

$$|F(ab) - F(a)F(b)| \leq \delta \|a\| \|b\|, \quad \text{for } a, b \in \mathcal{A}.$$

The investigation of these concepts was initiated approximately 30 years ago by B. E. Johnson and R. Rochberg. Johnson took an algebraic approach and related the stability of Banach algebras under small deformations to the cohomology groups. Rochberg took a more geometrical approach relating small deformations of algebras of analytic functions to quasiconformal deformations of the domains.

We discuss the history as well as more recent developments in this area and the relations to other problems including the existence of an "almost corona" and the multidimensional version of the Riemann Mapping Theorem. While we have fairly clear understanding of the general structure of small deformations many specific and fundamental questions remain open.