
(ii) Suppose now that k has the Pick property and $\{\lambda_i\}$ is an interpolating sequence for \mathcal{H}_k , so there are constants $c_1, c_2 > 0$ such that

$$c_1 \sum |a_i|^2 \leq \left\| \sum a_i g_i \right\|^2 \leq c_2 \sum |a_i|^2.$$

Let (w_i) be any sequence in the closed unit ball of l^∞ . Define a linear operator R on $\vee\{g_i\}$ by

$$R : g_i \mapsto \bar{w}_i g_i.$$

Then $\|R\| \leq \sqrt{\frac{c_2}{c_1}}$, because

$$\begin{aligned} \left\| \left(\frac{c_2}{c_1} - R^* R \right) \sum a_i g_i \right\|^2 &= \sum \bar{a}_i a_j \left\langle \left(\frac{c_2}{c_1} - R^* R \right) g_j, g_i \right\rangle \\ &= \sum \bar{a}_i a_j \left[\frac{c_2}{c_1} \langle g_j, g_i \rangle - w_i \bar{w}_j \langle g_j, g_i \rangle \right] \\ &= \frac{c_2}{c_1} \left\| \sum a_i g_i \right\|^2 - \left\| \sum a_i \bar{w}_i g_i \right\|^2 \\ &\geq \frac{c_2}{c_1} c_1 \sum |a_i|^2 - c_2 \sum |a_i|^2 \\ &\geq 0. \end{aligned}$$

Therefore by the Pick property, there is a multiplier ϕ of norm at most $\sqrt{\frac{c_2}{c_1}}$ such that $\phi(\lambda_i) = w_i$ on any finite subset of the $\{\lambda_i\}$. Passing to a weak-star limit, one gets that this holds on the whole sequence $\{\lambda_i\}$. As the sequence (w_i) is arbitrary, $\{\lambda_i\}$ is an interpolating sequence.