203: Homework 10 Due November 10

1. Let $A = (1, 2, 3)$, $B = (2, 3, 4)$, $C = (5, 7, 9)$. Find $x, y \in \mathbb{R}$ so that $C = xA + yB$.

2. Fina all real t so that $(1 + t, 1 - t)$ and $(1 - t, 1 + t)$ are linearly independent.

3. Let $\mathbf{i}, \mathbf{j}, \mathbf{k}$ and $\mathbf{i} + \mathbf{j} + \mathbf{k}$ be four vectors in $\mathbb{R}^3$. Show that any three are linearly independent, but all 4 are linearly dependent.

4. Find two bases for $\mathbb{R}^3$ containing the vectors $(1, 1, 2)$ and $(1, 0, 1)$.

5. Let L be the line in $\mathbb{R}^3$ through the points $(-3, 1, 1)$ and $(1, 2, 7)$. Determine which of the following points are on the line:
a) (-7,0,5)
b) (-7,0,-5)
c) (-11,1,11)

6. Let L be the line in $\mathbb{R}^2$ given by

$$\{X \in \mathbb{R}^2 : X \cdot N = P \cdot N\},$$

where P is on the line and N is a non-zero vector normal to the line. Let Q be a point in $\mathbb{R}^2$. Prove that the distance of Q to L is

$$\frac{|(P - Q) \cdot N|}{\|N\|}.$$