1. A point moves in space according to the vector equation

$$\mathbf{r}(t) = 4\cos t\mathbf{i} + 4\sin t\mathbf{j} + 4\cos t\mathbf{k}.$$

(a) Show that the path is an ellipse and find a Cartesian equation for the plane containing this ellipse.
(b) Show that the radius of curvature is $\rho(t) = 2\sqrt{2}(1+\sin^2(t))^{3/2}$.

2. Let a curve be given by

$$\mathbf{r}(t) = (e^t, e^{-t}, \sqrt{2}t).$$

Show that the curvature is given by

$$\kappa(t) = \frac{\sqrt{2}}{(e^t+e^{-t})^2}.$$

3. If a point moves so that the velocity and acceleration vectors always have constant lengths, prove that the curvature is constant at all points of the path. Express this constant in terms of $\|\mathbf{a}\|$ and $\|\mathbf{v}\|$.

4. If a curve is given by the polar equation $r = f(\theta)$, for $a \le \theta \le b \le a + 2\pi$, prove that its arc length is

$$\int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta.$$

5. . The curve described by the polar equation $r = a(1+\cos\theta)$, where $a > 0$ and $0 \le \theta \le 2\pi$, is called a cardiod. Draw a graph of the cardiod $r = 4(1+\cos\theta)$ and compute its arc length.