

1. Let  $p, n \in \mathbb{N}^+$ . Prove that

$$n^p < \frac{(n+1)^{p+1} - n^{p+1}}{p+1} < (n+1)^p.$$

2. Using 1, prove that

$$\sum_{k=0}^{n-1} k^p < \frac{n^{p+1}}{p+1} < \sum_{k=0}^n k^p.$$

3. Let  $f$  be integrable on  $[a, b]$ . Prove that for every  $c \in \mathbb{R}$

$$\int_{a+c}^{b+c} f(x-c) dx = \int_a^b f(x) dx.$$

4. Let  $f$  be integrable on  $[a, b]$ . Prove that for every  $k \in \mathbb{R} \setminus \{0\}$

$$\frac{1}{k} \int_{ka}^{kb} f\left(\frac{x}{k}\right) dx = \int_a^b f(x) dx.$$

5. Calculate

$$\int_0^2 (x-1)^2 dx,$$

using just the results we have proved so far.