203: Homework 6 Due: 6/October

1. Assume $\lim_{x\to a} f(x)$ exists. Prove that if $c \in \mathbb{R}$, then $\lim_{x\to a} cf(x) = c\ \lim_{x\to a} f(x)$.

2. Assume $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, and $M \neq 0$. Prove that

$$\lim_{x\to a} \frac{f(x)}{g(x)} \ = \ L/M.$$

3. Assume f is an even function on $[-b, b]$ and that f is integrable. Prove that

$$\int_{-b}^{b} f(x)dx \ = \ 2\int_{0}^{b} f(x)dx.$$

4. Assume g is an odd function on $[-b, b]$ and that g is integrable. Prove that

$$\int_{-b}^{b} g(x)dx \ = \ 0.$$

5. We proved in class that the Fibonacci numbers are given by the formula

$$F_n \ = \ \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Evaluate

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n}.$$

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6. How large must n be to ensure that $\dfrac{F_{n+1}}{F_n}$ is within 10^{-1} of its limit? Within 10^{-2}? Within 10^{-k}?