203: Homework 9 Due November 3

1. Prove that $f(x) = 2x$ is uniformly continuous on $\mathbb{R}$, and $g(x) = x^2$ is not.

2. Prove that if $I$ is an interval and $f : I \to \mathbb{R}$ satisfies:

$$\exists M \in \mathbb{R} \text{ such that } |f'(x)| \le M \ \forall \ x \in \ I, \tag{1.1}$$

then $f$ is uniformly continuous on $I$.

3. Give an example to show that the condition in (1.1) is not necessary for $f$ to be uniformly continuous.

4. Let $\mathbf{a} = (1,1,1),\ \mathbf{b} = (0,1,1),\ \mathbf{c} = (1,1,0)$ be three vectors in $\mathbb{R}^3$. Let $\mathbf{d} = x\mathbf{a} + y\mathbf{b} + z\mathbf{c}$.
(i) Determine the components of $\mathbf{d}$.
(ii) If $\mathbf{d} = 0$, prove that $x = y = z = 0$.
(iii) Find $x, y, z$ so that $\mathbf{d} = (1, 2, 4)$.
(iv) Calculate $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$.

5. Let $\mathbf{a} = (2,1,1,1),\ \mathbf{b} = (0,1,1,-1),\ \mathbf{c} = (1,-1/2,-1/2,3/2)$ be three vectors in $\mathbb{R}^4$. Let $\mathbf{d} = x\mathbf{a} + y\mathbf{b} + z\mathbf{c}$.
(i) Determine the components of $\mathbf{d}$.
(ii) If $\mathbf{d} = 0$, must $x = y = z = 0$?
(iii) Find $x, y, z$ so that $\mathbf{d} = (6, 1, 1, 5)$.
(iv) Calculate $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$.

6. Let $\mathbf{a} = (1,2,3,4)$ and $\mathbf{b} = (1,1,1,1)$. Calculate the orthogonal projection of $\mathbf{a}$ along $\mathbf{b}$, the orthogonal projection of $\mathbf{b}$ along $\mathbf{a}$, and the angle between $\mathbf{a}$ and $\mathbf{b}$.

7. Three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in $\mathbb{R}^5$ satisfy

$$\|\mathbf{a}\| = \|\mathbf{c}\| = 5, \quad \|\mathbf{b}\| = 1, \quad \|\mathbf{a} - \mathbf{b} + \mathbf{c}\| = \|\mathbf{a} + \mathbf{b} + \mathbf{c}\|.$$

If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/8$, what is the angle between $\mathbf{b}$ and $\mathbf{c}$?