204: Homework 1   Due: 26/January

Turn in perfect solutions to all the Final Exam questions from 203 for which you did not get full credit (10 pts).

For ease of grading, state which ones you did get full credit for.

1. Calculate the arc-length between $t = 0$ and $t = 2$ of $\mathbf{r}(t) = (12t, 8t^{3/2}, 3t^2)$.

2. a) If a particle moves with constant speed, prove that the acceleration is always perpendicular to the velocity.
b) Is the converse true? If so, prove it; if not, give a counter-example.

3. Prove that $f(x) = 3x$ is uniformly continuous on $\mathbb{R}$, and $g(x) = x^2$ is not.

4. Prove that the equation $x^2 = x \sin x + \cos x$ has exactly two real solutions.

5. Suppose I is an open interval in $\mathbb{R}$, that $f : I \to \mathbb{R}$, and $c \in I$. Suppose that for every sequence (x_n) in I that converges to c, $\lim_{n\to\infty} f(x_n)$ exists. Prove that all the limits are equal, and that $\lim_{x\to c} f(x)$ exists and equals the same number.

6. Prove that $\sqrt{12}$ is irrational.

7. Define the Fibonacci numbers by $F_1 = 1, F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Prove that F_{4n} is divisible by 3 for every $n \geq 1$.

8. a) Prove that every continuous function $f : [3, 4] \to \mathbb{R}$ attains its maximum.
b) Give an example of a continuous function $g : [3, 4] \cap \mathbb{Q} \to \mathbb{Q}$ that does not attain its supremum.