204: Homework 10 Due Thursday April 7

1. Evaluate the double integral $\int\int_R x^2y(x-y)dxdy$, where $R = [0,2] \times [0,1]$.

2. Evaluate the double integral of $x^2y(x-y)$ over each of the two triangles obtained by bisecting R from Problem (1) by the diagonal from $(0,0)$ to $(2,1)$.

3. Evaluate $\int\int_D x^2y^2dxdy$, where R is the bounded region in the first quadrant lying between the hyperbolas $xy = 1$ and $xy = 2$ and the lines $y = 2x$ and $y = 4x$.

4. A pyramid is bounded by the three coordinate planes and the plane $x + 2y + 4z = 7$. Find its volume.

5. Using Green's theorem, calculate the work done by the force $F(x,y) = (x^2 - y^2)\mathbf{i} + 2xy\mathbf{j}$ in moving a particle in the counterclockwise direction around the square with corners $(0,0), (a,0), (a,a), (0,a)$. (The work is the line integral of the force).

6. Use Green's theorem to calculate the integrals $\int_C y^2dx + 2xdy$, where C is:
(a) The square with vertices $(\pm 1, \pm 1)$.
(b) The circle of radius 1 centered at the origin.
(c) The positively oriented boundary of the annulus $\{(x,y) : 1 < x^2+y^2 < 4\}$.