204: Homework 11 Due Tuesday April 12

1. Evaluate $\int\int_R \cos(x^2+y^2)dxdy$, where R is the region above the x-axis and within the circle $x^2+y^2=9$.

2. Find the volume of the solid inside the cylinder $x^2+y^2=4$ and the ellispoid $4x^2+4y^2+z^2=64$.

3. A cylindrical drill with radius r_1 is used to bore a hole through the center of a sphere of radius r_2.

(a) Find the volume of the ring-shaped solid that remains.

(b) Express the volume in part a in terms of the height of the ring.

4. Calculate $\int\int_R \frac{1}{x+y}dxdy$, where R is the region bounded by $x=0, y=0, x+y=1, x+y=4$, using the change of variable $T(u,v)=(u-uv,uv)$.

5. Find the center of mass of the region D between $y=x^2$ and $y=x$ if the density is $\rho(x,y)=x+y$.

Note: The mass is $m=\int\int_D \rho(x,y)$. The center of mass is then $(\bar{x},\bar{y})$, where

$$\begin{aligned}\bar{x} &= \frac{1}{m}\int\int_D x\rho(x,y)\\ \bar{y} &= \frac{1}{m}\int\int_D y\rho(x,y).\end{aligned}$$