204: Homework 12 Due Tuesday April 19

1. Evaluate $\int\int\int_D z dV$ where D is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0, y = 3x, z = 0$ in the first octant.

2. Evaluate $\int\int\int_D yz\cos(x^5)dV$, where

$$D = \{(x, y, z) : 0 \leq x \leq 1,\ 0 \leq y \leq x,\ x \leq z \leq 2x\}.$$

3. Find the mass and center of mass of the solid E bounded by the paraboloid $z = 4x^2 + y^2$ and the plane $z = 2$, assuming it has constant density.

4. Find the volume of the solid that lies within the sphere $x^2+y^2+z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

5. Find the volume of the regular tetrahedron with side 1.

6. The stem of a mushroom is a cylinder of diameter 1 and height 2, and its cap is a hemisphere of radius R. If it is constant density, and its center of mass is in the plane where the stem joins the cap, find R.

7. For $a > 0$, let $O_n(a)$ be the set in $\mathbb{R}^n$ given by

$$O_n(a) = \{(x_1, x_2, \ldots, x_n) : |x_1| + |x_2| + \cdots + |x_n| \leq a\}.$$

Let $V_n(a)$ be the volume of $O_n(a)$.
(i) Prove that $V_n(a) = a^n V_n(1)$.
(ii) Prove that

$$V_n(1) = V_{n-1}(1)\int_{-1}^{1}(1 - |x|)^{n-1}dx.$$

(iii) Deduce that $V_n(a) = \frac{2^n a^n}{n!}$.