204: Homework 14 Not to be turned in. One question will be on final

1. Use the Divergence theorem to calculate the flux of $\mathbf{F}$ across the surface Σ, where $F(x, y, z) = 4x^3z\mathbf{i} + 4y^3z\mathbf{j} + 3z^4\mathbf{k}$ and Σ is the sphere with radius R centered at the origin.

2. Use the Divergence theorem to calculate the flux of $\mathbf{F}$ across the surface Σ, where $F(x, y, z) = z^2x\mathbf{i} + (\frac{1}{3}y^3 + \sin z)\mathbf{j} + (x^2z + y^2)\mathbf{k}$, and Σ is the top half of the sphere $x^2 + y^2 + z^2 = 1$. (Note: Σ is not a closed surface, so you must adjoin a disk to it).

3. Use Stokes's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $F(x, y, z) = e^{-x}\mathbf{i} + e^x\mathbf{j} + e^z\mathbf{k}$, and C is the boundary of the plane $2x + y + 2z = 2$ in the first octant.

4. Verify that Stokes's theorem is true by calculating both sides for the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + xyz\mathbf{k}$, and Σ is the part of the plane $2x + y + z = 2$ in the first octant, oriented upwards.