## 204: Homework 2 Due February 2

1. Prove that matrix multiplication is associative.
2. Let $f: X \rightarrow Y$. Prove that $f$ has a left-inverse if and only if it is injective. Prove that it has a right inverse if and only if it is surjective.
3. Suppose $\lim _{n \rightarrow \infty} x_{n}=L$. Prove that every subsequence of $\left(x_{n}\right)$ also converges to $L$.
4. Find two 2-by-2 matrices $A$ and $B$ that are not linearly dependent, have every entry non-zero, and satisfy $A B=B A$.
5. For each of the following matrices, find its rank and its nullity.

$$
\begin{gathered}
\left(\begin{array}{ll}
1 & 2 \\
3 & 4
\end{array}\right) \\
\left(\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}\right) \\
\left(\begin{array}{ccc}
1 & 2 & -1 \\
3 & 4 & 1
\end{array}\right) \\
\left(\begin{array}{cc}
1 & 3 \\
2 & 4 \\
-1 & 1
\end{array}\right)
\end{gathered}
$$

