

204: Homework 3 Due February 9

1. Find the eigenvalues and eigenvectors for the matrices

$$\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}.$$

2. Find the eigenvalues and eigenvectors for the matrix

$$\begin{pmatrix} 3 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

3. Let S be the subset of \mathbb{R}^2 given by $\left\{ \begin{pmatrix} x \\ y \end{pmatrix} : xy = 1 \right\}$. Is this open, closed, or neither? Prove your answer.

4. Let S be the subset of \mathbb{R}^2 given by $\left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x \notin \mathbb{Z}, y \notin \mathbb{Z} \right\}$. Is this open, closed, or neither? Prove your answer.

5. Let K be a closed bounded set in \mathbb{R}^m . Prove that every sequence has a subsequence that converges to a point of K . (Hint: First find a subsequence whose first coordinate converges. Then find a subsequence of that whose second coordinate converges, and so on.)