204: Homework 4 Due February 16

1. Calculate the partial derivatives of the functions

$$\begin{aligned} f(x,y) &= x^3 - 3x^2y + 2xy^3 + 5, \\ g(x,y,z) &= xyz \\ h(x_1,\ldots,x_n) &= \sum_{j=1}^{n} x_j^2. \end{aligned}$$

2. For the functions in the previous question, calculate the directional derivative at $\mathbf{a}$ in the direction $\mathbf{v}$ where $\mathbf{a}$ and $\mathbf{v}$ are, respectively, $(1,2)^t$, $\frac{1}{\sqrt{10}}(1,3)^t$; $(1,-1,1)^t$, $\frac{1}{\sqrt{13}}(1,2,3)^t$; and $(1,1,\ldots,1)^t, (\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}, \ldots, \frac{1}{\sqrt{n}})^t$.

3. Let $f(x,y) = \frac{xy}{x^2+y^2}$ when $(x,y) \neq 0$, and $f(0,0) = 0$. Show that the partial derivatives of f exist at 0 but the function is not continuous at 0. Do other directional derivatives of f exist at 0?

4. Find the equation of the tangent plane of the graph of f at the given point:

$$\begin{aligned} f(x,y) = x^2y^3 - 2y; &\qquad \mathbf{a} = (2,-1)^t \\ f(x,y) = x^2 + y^2; &\qquad \mathbf{a} = (2,-1)^t \\ f(x,y,z) = x^2yz; &\qquad \mathbf{a} = (2,-1,3)^t. \end{aligned}$$

5. Give the derivative matrix for the following functions:

$$\begin{aligned} f(t) &= \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix} \\ f\begin{pmatrix} r \\ \theta \\ z \end{pmatrix} &= \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ z \end{pmatrix} \\ f\begin{pmatrix} \rho \\ \theta \\ \phi \end{pmatrix} &= \begin{pmatrix} \rho\sin\phi\cos\theta \\ \rho\sin\phi\sin\theta \\ \rho\cos\phi \end{pmatrix}. \end{aligned}$$