204: Homework 5 Due February 23

1. Prove that a differentiable function from an open set in $\mathbb{R}^n$ to $\mathbb{R}^m$ is continuous.

2. Let K be a closed bounded set in $\mathbb{R}^n$, and $f : K \to \mathbb{R}$ be continuous. Prove that f is bounded, and f attains its minimum and maximum. (You may want to use the result of HW 3, Problem 5).

3. Let f, g be differentiable functions from $\mathbb{R}^n$ to $\mathbb{R}^m$. Prove that $D(f + g)(a) = Df(a) + Dg(a)$.

4. Let $f(x,y) = \frac{xy}{x^2+y^2}$ when $(x,y) \neq 0$, and $f(0,0) = 1/4$. Which directional derivatives of f exist at 0?

5. Let

$$\begin{aligned} f(x,y) &= (1 + 2x^2 - 3xy, 2 - 4x + 5y^2)^t, \\ g(x,y) &= (2 - x - xy, x^2y^2)^t, \\ k(x,y) &= 4 - 2x - 5y - 7xy^2. \end{aligned}$$

Calculate their derivatives at the point $(2,1)$. Calculate the derivatives of $f \cdot g$ and kf at $(2,1)$ using the product rule.