204: Homework 6 Due March 1

1. An ant moves along a helical path

$$\mathbf{r}(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ 3t \end{pmatrix}.$$

(a) At what rate is her distance from the origin changing when $t = 0$? When $t = 2\pi$?

(b) If the temperature is given by $T(x, y, z) = xy + z + z^2$, at what rate is the temperature changing for the ant when $t = \pi/4$?

2. Let U be open in $\mathbb{R}^n$, and $f : U \to \mathbb{R}$ be differentiable. Let $a \in U$, and assume $f(a) \neq 0$. Prove that $1/f$ is differentiable at a, and find a formula for its derivative.

3. Find the equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 14$ at the point $(1, 2, 3)$.

4. Let

$$f(x, y) = xy\frac{x^2 - y^2}{x^2 + y^2}, \quad (x, y) \neq 0$$

and $f(0, 0) = 0$. Calculate its second order partial derivatives everywhere. Is the function C^2?

5. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a differentiable function whose gradient is never 0 and satisfies

$$\frac{\partial f}{\partial x} = 2\frac{\partial f}{\partial y}$$

everywhere. Find the level curves of f.