204: Homework 7 Due March 8

1. A rectangular box with edges parallel to the coordinate axes has one corner at the origin, and the opposite corner on the plane $x + 2y + 4z = 7$. What is the maximum possible volume of the box?

2. A rectangular box is inscribed in a hemisphere of radius 1. What is its maximum volume?

3. The temperature of a circular plate, $\{\mathbf{x} \in \mathbb{R}^2 : \|x\|^2 \leq 2\}$, is given by $f(x, y) = x^2 + 2y^2 - 2x$. Find the maximum and minimum temperature on the plate.

4. Suppose x, y, z are positive numbers and $xy^2z^3 = 108$. Find the minimum value of their sum.

5. Consider the function $f(x, y) = 2x^4 - 3x^2y + y^2$.
(a) Show that the origin is a critical point.
(b) Show that on any line through the origin the origin is a local minimum point.
(c) Is the origin a local minimum of f?

6. For the following functions, find and classify all their critical points:
(a) $f(x, y) = x^2 + 3x - 2y^2 + 4y$.
(b) $f(x, y) = \sin x + \sin y$.
(c) $f(x, y, z) = xyz - x^2 - y^2 - z^2$.
(d) $F(x, y, z) = x^3 + xz^2 - 3x^2 + y^2 + 2z^2$.