350: Midterm March 1

1. (i) Consider the 1-dimensional system $\dot{x} = f(x)$ where f looks like the picture. What are the fixed points, and which ones are attracting, and which ones are repelling?

(ii) If f is changed to $f(x) + \varepsilon$, where ε is a small real number (which may be positive or negative) how does the answer change?

2. (i) Sketch the trajectories of a 2-dimensional linear system whose fixed point is a center.

(ii) What could happen qualitatively if there were higher order non-linear terms?

(iii) If you knew that the system was conservative, how would that change the answer to (ii)?

3. (i) Consider the 2-dimensional system

$$\begin{aligned}\dot{x} &= x + 2y \\ \dot{y} &= 3x + 4y\end{aligned}$$

What is the nature of its fixed point?

(ii) What could happen qualitatively to the fixed point at 0 if one looked at the system

$$\begin{aligned} \dot{x} &= x + 2y + xy \\ \dot{y} &= 3x + 4y - y^2 \end{aligned}$$

4. (i) What is the index of a closed curve with respect to a vector field $\vec{F}(x, y)$?

(ii) What is the index of the vector field

$$\vec{F}(x, y) = \begin{pmatrix} x^2 \\ x - y \end{pmatrix}$$

at $(0, 0)$?

5. Consider the damped harmonic oscillator

$$\ddot{x} + b\dot{x} + x = 0$$

where $b \geq 0$. Analyze the system and describe it qualitatively as b ranges from 0 to $+\infty$.