350: Homework 1   Due: 1/February

1. Consider the ODE

$$P'(t) = P(P-1)(5-P).$$

Sketch solutions for different initial conditions. What are the fixed points? Which are attracting and which are repelling? If this modeled a biological population, what are the qualitative features of the model.

2. For both of the next two equations, sketch the vector field (as in Example 2.8.1 in Strogatz), find all the fixed points, classify their stability, and sketch the graph of x for different initial conditions.
(i) $\dot{x} = 1 + \frac{1}{2}\cos(x)$
(i) $\dot{x} = 1 + 2\cos(x)$

3. Do problem 2.2.8

4. Do problem 2.2.9

Computer Exercises

You need to learn to use a program that solves ODE's. There are several, such as Mathematica and Matlab. I will give explicit instructions for Octave, which is free, but you may use other packages if you prefer. The purpose of the next 2 exercises is just to get a package installed and to run it with simple examples. Play around with parameters (like the number of t-values) to see what happens.

C1. Plot solutions of the ODE from exercise 1 numerically.

Here is one way to do it in Octave GUI.

Have a function file called LogThresh.m. (Notice it must have the same name as the function). It would contain the following:

```
function xdot = LogThresh(x,t)
c1 = 1;
c2 = 5;
xdot = x*(x-c1)*(c2-x);
endfunction
```

Now have a command file called RunLogThresh.m, containing

```
x1 = 1.19;
t=linspace(0,3,100)';
x=lsode("LogThresh",x1,t);
plot(t,x)
```

Alternatively, you could enter all these lines on a command window; the advantage of saving scripts is it is easier to modify them.

C2. Consider the pendulum equation

$$\ddot{\theta} = -\sin(\theta).$$

Solve this with initial conditions $\theta(0) = 0, \dot{\theta}(0) = 1.99$ and $\theta(0) = 0, \dot{\theta}(0) = 2.01$. What is the qualitative difference?

Here is my Octave code. Now x is a 2-dimensional vector, with $x(1) = \theta$ and $x(2) = \theta'$.

For the function, saved in Pendulum.m

```
function xdot = Pendulum(x,t)
c= -1;
xdot(1) =x(2);
xdot(2) = c*sin(x(1));
endfunction
```

And to run it

```
x0 = [0,2.0];
t=linspace(0,50,250)';
x=lsode("Pendulum",x0,t);
plot(t,x)
```