

350: Homework 10 Due: 19/April

1. Prove that every continuous function $f : [0, 1] \rightarrow [0, 1]$ has a fixed point.

2. Define $f : [0, 1] \rightarrow [0, 1]$ by

$$\begin{array}{lcl} 0 & \mapsto & \frac{1}{2} \\ \frac{1}{4} & \mapsto & 1 \\ \frac{1}{2} & \mapsto & \frac{3}{4} \\ \frac{3}{4} & \mapsto & \frac{1}{4} \\ 1 & \mapsto & 0, \end{array}$$

and letting f be piecewise linear in between. Show that f has no period 3 cycles.

3. Calculate the exact perimeter of a regular n -gon inscribed in the unit circle. Hence show that the dimension of the circle, in the sense of Richardson, is 1.

4. Do 11.3.1.

5. Do 11.3.8.

6. Do 11.4.6.

Bonus problems - don't hand in with the assignment. These are just for practice. (You are welcome to give them to me to grade, or to talk about them with me in my office hours. But they carry no credit).

Conceptual:

Prove that the Hausdorff dimension of the circle is exactly 1.