As an explanation of how chance can arise in a world which he regarded as strictly deterministic, Henri Poincaré [183] drew attention to insignificant causes which produced very noticeable effects. Seacoasts provide an apt illustration. For the spike of the dividers may just miss, or just catch, a promontory of land or the head of a loch; so that an insignificant change in l may alter Σl noticeably.

The west coast of Britain from Land's End to Duncansby Head was chosen as an example of a coast that looks more irregular than most other coasts in an atlas of the world. The quality and scale of the maps of Britain were such that the irregularities of the coast greatly predominated over any errors that are likely to have occurred in the processes of drawing, printing, or reading the maps. For the longer steps the *Times Atlas 1900* was used. The 10 kilometre steps were counted on the *British Isles Pocket Atlas 1935* by John Bartholemew FRGS. Narrow waters were regarded as barred by any bridges shown in the latter atlas. The Mersey tunnel was also regarded as a barrier, but the Severn tunnel was disregarded. These rules were kept the same for all lengths of step. The over-all length in one step was found to be 971 km. The attempt to make exactly two equal steps is worth stating in detail because it illustrates principles and peculiarities. The map was page 15 of the *Times Atlas* dated 1900. A circle with centre at Duncansby Head and radius 490 km cuts the coast of Cumberland

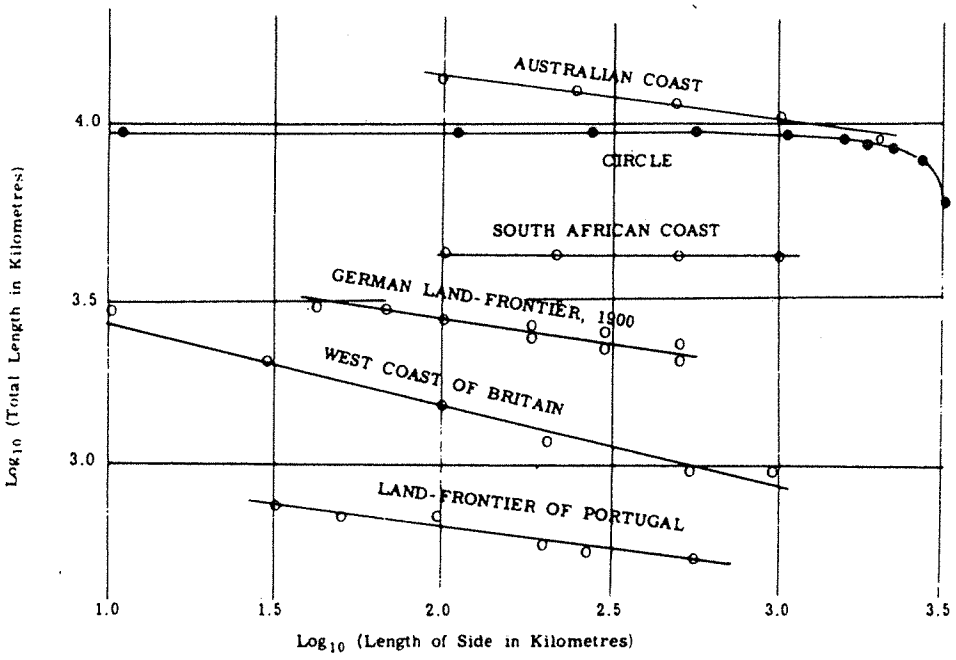

Figure 9.4: Measurements of curves by way of polygons (not shown) which have equal sides and have their corners on the curve. The slope of a graph shows how the total length of the polygon increases as its side becomes shorter. The convergence for the circle contrasts with the behaviour for frontiers.

partly by mountains, and elsewhere by neither, I chose that of Germany as shown in the *Times Atlas* dated 1900, pp.39–40. Since then the frontier has been much altered, but previously it had persisted from 1871. [The results are given in Table 9.14]. Again the total length depends on whether the start is at the east or west. Nevertheless, for a conspectus, a straight line on the graph may serve.

9.3.1 Conclusions on polygons of equal sides inscribed to frontiers

1 The fitting of a whole number of sides can only be done by troublesome successive approximations.
2 It is occasionally impossible to fit a whole number of sides.
3 The possibility of fitting a whole number of sides occasionally depends on which end of the frontier is taken as the starting point.
4 For the above three reasons it is preferable to allow the final side to be an estimated fraction of the standard side.
5 The total polygonal length, including the estimated fraction, usually depends slightly on the point that is taken as the start.
6 It is doubtful whether the total polygonal length of a seacoast tends to any limit as the side of the polygon tends to zero.
7 To speak simply of the 'length' of a coast is therefore to make an

Table 9.12. *Land-frontiers.*

Land-frontier between	Kilometres as stated by:	
	the former country	the latter country
Spain and Portugal	987	1214
Netherlands and Belgium	380	449.5
USSR and Finland	1590	1566
USSR and Romania	742	812
USSR and Latvia	269	351
Estonia and Latvia	356	375
Yugoslavia and Greece	262.1	236.8

Table 9.13. *The land-frontier between Spain and Portugal.*

Start	S or NW	S or NW	S	NW	S	NW	S	NW	S	NW
No. of sides	1	2	3	—	7.05	7.07	13.10	13.06	27.2	27.05
Side, km	543	285	201	—	100	100	56.29	56.36	30	30
Total, km	543	570	603	—	705	707	737	736	816	812