How Often Do You Want Your Team to Win?

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When I daydream about living in Heaven, I imagine playing rugby every Saturday afternoon. The teams will not need substitutes, because nobody ever gets injured. My team will win most of the time (as will everybody else's—thanks to heavenly temporal anomalies). How often should my team win to maximize my happiness?

Let p be your team's long-term winning percentage. Let $W(p)$ be the pleasure you experience when your team wins a game. The lower p is, the sweeter the victory; so we can assume that $W(p)$ decreases as p increases, and $W(1) = 0$, since there is (or should be) no pleasure in a guaranteed win. In the words of Manchester City's manager Pep Guardiola, "It's not a sport if you can't lose."

How rapidly $W(p)$ decreases can vary from person to person. Let us assume that

$$W(p) = 1 - ap - bp^2 - cp^3,$$

where a, b, c are constants chosen so that $W(1) = 0$.

We will denote the pain you experience if your team loses a game by $L(p)$. We shall assume by symmetry that $L(p) = W(1-p)$. So the overall expected satisfaction you get from your team is

$$S(p) = pW(p) - (1-p)L(p).$$

The first term on the right is the pleasure you get from winning multiplied by the probability of a win, and the second is the pain from losing multiplied by the probability of a loss.

The surprise is that no matter how you choose a, b, and c, subject to $W(1) = 1 - a - b - c = 0$, the corresponding satisfaction function $S(p)$ attains its extrema at the same values of p. Indeed, some algebra shows that

$$S(p) = (b + 2c)\left(3p^2 - p - 2p^3\right).$$

The extreme values occur where the derivative of S is zero, which happens at $\frac{1}{2} \pm \frac{\sqrt{3}}{6}$. So as long as $b + 2c \geq 0$ (which corresponds to $W(p)$ being above the orange curve in Figure 1 as p tends to 1), the graphs look like the curves in Figure 2. The maximum satisfaction occurs, then, at

$$p_h = \frac{1}{2} + \frac{\sqrt{3}}{6} \approx 78.9\%,$$

and the most pain occurs if your team's winning rate is

$$p_u = \frac{1}{2} - \frac{\sqrt{3}}{6} \approx 21.1\%.$$

What team comes closest to the heavenly win rate? In 2018, Connor Fleming calculated[1] that the most successful—in terms of winning percentage—professional sports teams by sport are as shown in Table 1.

This should make supporters of the All Blacks (the New Zealand Men's Rugby team) the happiest fans in the world (even though their current winning record has slipped to 77%).

Notes

If you count a tie as half a win and half a loss, you want the sum of your winning probability plus half the probability of a tie to equal p_h.

Figure 2 shows that people whose win satisfaction is more concave (closer to the blue curve) get more satisfaction as long as their team wins more than it loses.

The fact that all the curves in Figure 2 have extrema at the same points is due to them all being multiples of the same cubic polynomial. If you allow for even more concavity in $W(p)$ (the greater the concavity, the more pleasure you get when your team beats a much weaker opponent), then the optimal winning percentage will change. For example,

[1]See https://the18.com/en/soccer-entertainment/the-winningest-team-in-sports-history.

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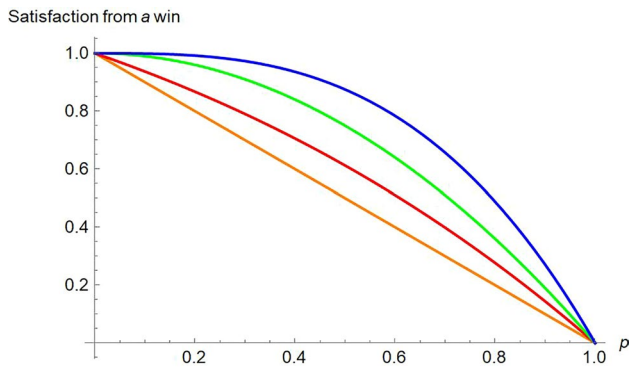

Figure 1. Different choices of W, from $1-p$ (orange) to $1-p^3$ (blue).

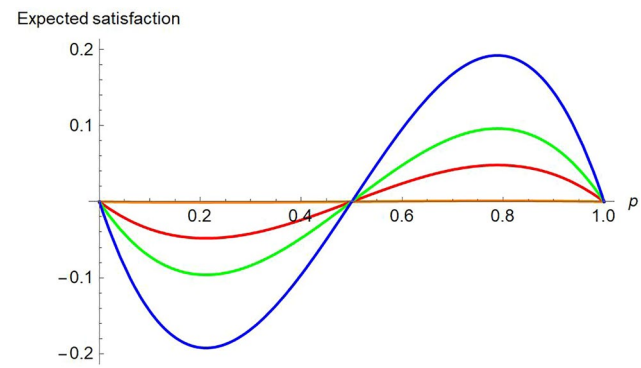

Figure 2. Expected satisfaction S for different choices of W. The colors represent the same values of W as in Figure 1.

Table 1. Six professional sports with their highest winning percentages

Sport	Team	Winning Percentage
Men's International Basketball	USA	88.6%
Women's International Soccer	USA	84.0%
Men's International Rugby	New Zealand	79.2%
Men's International Soccer	Brazil	73.5%
English Premier League	Manchester United	73.5%
Bundesliga	Bayern München	70.3%

if $W(p) = 1 - p^4$, then the optimal winning percentage edges up to 79.4%. But surely all readers of the *Mathematical Intelligencer* are more fair-minded than that!

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