MAT 132

Final Exam Spring 2017

Name: ______________________ **ID:** ______________________

Recitation: ______________________

List of Recitations:

R01	MW 12:00 pm-12:53 pm	Frederik Benirschke
R02	TuTh 10:00 am-10:53 am	Juan Ysimura
R03	TuTh 8:30am- 9:23am	Erik Gallegos Baños
R04	MW 11:00am-11:53am	Paul Frigge
R20	TuTh 5:30pm-6:23pm	Jean-François Arbour
R21	10:00am- 10:53am	Lisandra Hernandez Vazquez
R22	TuTh 11:30am- 12:23pm	Robert Abramovic

Instructions:

(1) Fill in your name and Stony Brook ID number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. $1/2$, not $2/4$) to receive full credit.

(4) Answer all questions in the space provided. If you need more room use the blank backs of the pages.

(5) Show your work; correct answers alone will receive only partial credit.

Problem	1 (16 pts)	2 (16 pts)	3 (16 pts)	4 (14 pts)	5 (16 pts)	6 (16 pts)	7 (16 pts)	8 (16 pts)	9 (14 pts)	Total (140 pts)
Score										

1. Compute the following integrals

   (a) $\displaystyle\int \frac{10x-19}{x^2-x-12}\,dx$

   (b) $\displaystyle\int_1^e \frac{(\ln(x))^2+1}{x}\,dx$

2. A large tank contains 50 kg of salt dissolved in 10000 liters of water. A solution of water and salt containing 0.02 kg of salt per liter enters the tank through a pipe at a rate of 3 liters per minute. At the same time, the the mixture of salt and water is pumped out of the tank at a rate of 3 liters per minute. The tank is kept well mixed and the concentration of salt in the tank is uniform. Let $S(t)$ denote the amount of salt in the tank after t minutes, measured in kg .

(a) Write down a differential equation for $S(t)$.

(b) Write down an initial condition for $S(t)$.

(c) Solve the differential equation with the initial condition given in parts (a) and (b).

3. (a) Find the interval of convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{5^n (x+1)^n}{\sqrt{n+4}}$$

(b) Find the Maclaurin series of $f(x) = xe^{\frac{1}{2}x^4}$. What is the radius of convergence?

4. (I) Which one of the following options gives a convergent improper integral?

(a) $\displaystyle\int_0^1 \frac{1}{x+1}\,dx$

(b) $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$

(c) $\displaystyle\int_1^\infty \frac{1}{x^2+1}\,dx$

(d) $\displaystyle\int_1^\infty \frac{1}{x+1}\,dx$

(II) We are looking for the Maclaurin series of a function $f(x)$. We know that $f^{(20)}(0) = 19! \cdot 2^{19}$. What is the coefficient of x^{20} in the Maclaurin series of $f(x)$?

(a) $20! \cdot 19! \cdot 2^{19}$ (b) $\dfrac{1}{20 \cdot 2}$ (c) $\dfrac{2^{19}}{20}$ (d) $19! \cdot 2^{19}$

Recall that the coefficient of x^n in a power series $\displaystyle\sum_{i=0}^{\infty} a_i x^i$ defined to be a_n.

(III) A power series is convergent at -1 and divergent at 1. Which one of the following points could be in the interval of the convergence of this power series:

(a) 2 (b) -2 (c) 4 (d) 10

(IV) Write down a convergent **geometric** series with infinitely many non-zero terms such that the sum is equal to e.

5. We want to use a Taylor polynomial of $f(x) = \sqrt[3]{x}$ to approximate $\sqrt[3]{25}$.

   (a) What would be a good choice for the center of the Taylor polynomial? Why?

   (b) Use the Taylor polynomial of degree 2 centered at the point that you found in the previous part to write an estimate for $\sqrt[3]{25}$.

   (c) Find an upper bound for the error of your estimate.

6. (a) Solve the following initial value problem:

$$y'' - 2y' + 5y = 0 \qquad y(\frac{\pi}{2}) = 0 \qquad y'(\frac{\pi}{2}) = 2$$

(b) Find the general solution of the following differential equation:

$$(x^2 + 1)y' + 3x^3 y = 6xe^{-\frac{3}{2}x^2}$$

7. (a) Determine whether the following series is convergent or divergent. If it converges, find its sum.

$$\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n)!3^{2n}}$$

(b) Determine whether the following series is convergent or divergent. State the test you use, and show your work:

$$\sum_{n=2}^{\infty} \frac{2+\cos(n^2)}{\sqrt{n}-1}$$

8. (a) Let $\mathcal{R}$ be the region enclosed by the x-axis, $x = 2$ and $y = x^2$. Find the volume obtained by rotating $\mathcal{R}$ about the line $y = -4$.

(b) Find the length of the curve $x(t) = \cos(t) + t\sin(t)$, $y(t) = \sin(t) - t\cos(t)$ for $0 \leq t \leq \pi$.

9. Find a positive number t such that the average of the function $f(x) = (x+1)e^x$ in the interval $[0, t]$ is equal to 2.

Scratch Paper