Midterm $2 \quad$ Spring 2017
Name: $\qquad$ ID: $\qquad$

## Recitation:

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## List of Recitations:

| R01 | MW 12:00 pm-12:53 pm | Frederik Benirschke |
| :--- | :--- | :--- |
| R02 | TuTh 10:00 am-10:53 am | Juan Ysimura |
| R03 | TuTh 8:30am- 9:23am | Erik Gallegos Baños |
| R04 | MW 11:00am-11:53am | Paul Frigge |
| R20 | TuTh 5:30pm-6:23pm | Jean-François Arbour |
| R21 | 10:00am- 10:53am | Lisandra Hernandez Vazquez |
| R22 | TuTh 11:30am- 12:23pm | Robert Abramovic |

## Instructions:

(1) Fill in your name and Stony Brook ID number at the top of this cover sheet.
(2) This exam is closed-book and closed-notes; no calculators, no phones.
(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not $\approx 1.4$ ) and simplify them as much as possible (e.g. $1 / 2$, not $2 / 4$ ) to receive full credit.
(4) Answer all questions in the space provided. If you need more room use the blank backs of the pages.
(5) Show your work; correct answers alone will receive only partial credit.
(6) This exam has 5 extra credit points.

| Problem | 1 <br> $(10 \mathrm{pts})$ | 2 <br> $(10 \mathrm{pts})$ | 3 <br> $(10 \mathrm{pts})$ | 4 <br> $(10 \mathrm{pts})$ | 5 <br> $(15 \mathrm{pts})$ | 6 <br> $(15 \mathrm{pts})$ | 7 <br> $(15 \mathrm{pts})$ | 8 <br> $(20 \mathrm{pts})$ | Total <br> $(105 \mathrm{pts})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Score |  |  |  |  |  |  |  |  |  |

1. (I) (2 points) Which one of the following options is correct?
(a) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ are both convergent.
(b) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ are both divergent.
(c) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is convergent and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
(d) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is divergent and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is convergent.
(II) (2 points) Of the following series listed below, select ALL which are geometric series. (There are exactly two correct answers.)
(a) $\sum_{n=1}^{\infty} \frac{e^{n}}{n}$
(b) $\sum_{n=1}^{\infty} 2^{\frac{1}{n}}$
(c) $\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^{n-1}}$
(d) $\sum_{n=1}^{\infty} n^{\frac{1}{2}}$
(e) $\sum_{n=1}^{\infty} e^{2 n+5}$
(III) (3 points) Give an example of a divergent series $\sum_{n=1}^{\infty} a_{n}$ such that $\sum_{n=1}^{\infty} a_{n}^{2}$ is convergent. Explain briefly why your series satisfies these two conditions.
(IV) (3 points) Write the number $3 . \overline{48}=3.484848 \ldots$ as the ratio of two integer numbers in a reduced form. (Just give a fraction as the final answer. You do not need to justify your answer.)

Determine whether the following three series converge or not. State the tests that you are using and show your work. (10 points for each series)
2. $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{2}}$
3. $\sum_{n=1}^{\infty} \frac{3 n^{3}}{2 n^{3}+3 n-1}$
4. $\sum_{n=1}^{\infty} \sqrt{\frac{n+2}{n^{4}}}$
5. (15 points) For each of the following improper integrals, determine whether it is convergent or not. If it is convergent, evaluate the integral:
(a) $\int_{1}^{\infty} x e^{-x^{2}} d x$
(b) $\int_{0}^{1} \frac{2+\sin (x)}{x^{3}} d x$
6. (15 points) The temperature of Stony Brook on March 29 can be modeled by the following function:

$$
T(t)=2+14 \sin ^{2}\left(\frac{\pi t}{24}\right)
$$

where $T(t)$ denotes the temperature $t$ hours after 12:00 am. (The temperature is measured in ${ }^{\circ} \mathrm{C}$.) Find the average temperature of Stony Brook on that day.
7. (15 points) Find the length of the arc which is parametrized as $x=t^{3}$ and $y=\frac{2}{3} t^{\frac{9}{2}}$ for $t \in[0,2]$.
8. (20 points) A flat plate with uniform density has the shape of the region enclosed by $y=\frac{\ln (x)}{\sqrt{x}}, x=e$, $x=e^{2}$ and $y=0$. Find the coordinates of the center of mass of the plate.

