Practice Exam for Midterm 1

Evaluate the following integrals:

1. $\int x \arctan(x)\, dx$

2. $\int_0^3 \frac{x^3 + x}{\sqrt{x^2 + 1}}\, dx$

3. $\int e^{3t}\cos(t)\,dt$

4. $\int \frac{\sqrt{x^2-4}}{x^3}\,dx$ (use the substitution $x = 2\sec(u)$)

5. $\int_1^2 t^2 \ln(t)\, dt$

6. $\int \frac{3x+2}{x^2+5x+4}\, dx$

7. Bobby's farm has the shape of the region enclosed by the parabola $y = x^2$ and the horizontal line $y = 4$. Bobby wants to divide his farm into two parts O and P by a line of the farm $y = h$. Find the value of h such that O and P have the same area.

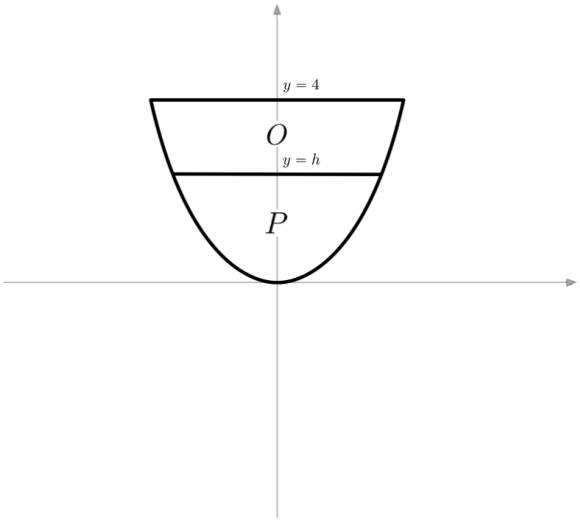

Figure 1: Bobby's farm

8. Let $\mathcal{R}$ be the region bounded by the y-axis, $y = \frac{1}{x}$, $y = 1$ and $y = 2$.

   (a) Sketch the shape of this region in the coordinate plane.

   (b) Let $\mathcal{S}$ be the solid given by rotating the region $\mathcal{R}$ about the vertical line $x = -1$. Find the volume of $\mathcal{S}$.

9. Let S be the region obtained by rotating the region enclosed by the x-axis and the curves $y = \ln(x^2)$, $x = e$ about the y-axis. In order to find the volume of S, firstly explain how you slice the region. Then write down the answer as an integral and evaluate the integral.