

Practice Exam for Midterm 2

1. Evaluate the sum of the following series:

$$\sum_{n=1}^{\infty} 2^{3n+2} 3^{-2-2n}$$

2. Find the length of the curve $y = \int_0^x \sqrt{t^2 + 6t + 8}$ for $1 \leq x \leq 4$.

3. Determine whether the following series converges or not. State the test that you are using and show your work.

$$\sum_{n=2}^{\infty} \sqrt{\frac{n-2}{n^5+2n+3}}$$

4. Determine whether the following series converges or not. State the test that you are using and show your work.

$$\sum_{n=4}^{\infty} \frac{1}{n \ln(n) [\ln(\ln(n))]^{1/2}}$$

5. Determine whether the following improper integral converges or not.

$$\int_0^1 \frac{1}{\sqrt{x} + x^2} dx$$

6. Find all the values of the parameter t such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, t]$ is 3.

7. Find the center of mass of a plate which has the shape of a quarter-circle of radius 2 and has uniform density. To answer this question, you firstly need to put the quarter-circle in a coordinate system. Please be explicit about what coordinate system you choose and then give your final answer with respect to your coordinate system.

8. A swimming pool, filled with water, has the shape of an inverted frustum. (A frustum is obtained from a right circular cone by cutting off the tip.) The radius of the upper and lower bases are respectively equal to 12 m and 8 m. The height of the pool is also equal to 2 m. Find the work required to empty the pool by pumping all of the water to the top of the pool. (The density of water is 1000kg/m^3 .)