Mat 211
Midterm 1 Spring 2016

Name: _____ **ID:** _____

Problem	1	2	3	4	5	6	7	8	Total
Score									

Instructions:

(1) Fill in your name and Stony Brook ID number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(4) Answer all questions in the space provided. If you need more room use the blank backs of the pages.

(5) Show your work; correct answers alone will receive only partial credit.

1. In each part, the augmented matrix of a linear system is given. Determine whether the solution is unique, there are no solutions or whether there are infinitely many. If the solution is unique give it. If there are infinitely many give the solution parametrically.

(i) $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 5 \\ 0 & 0 & 1 & 4 & 6 \end{array}\right]$

(ii) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right]$

2. Solve the following linear system using an augmented matrix. State whether the solution is unique, there are no solutions or whether there are infinitely many. If the solution is unique give it. If there are infinitely many give the solution parametrically.

$$\begin{cases} & & x_2 & + & 2x_3 & = & 3 \\ 2x_1 & - & 3x_2 & + & 2x_3 & = & -1 \\ 2x_1 & + & 3x_2 & + & 14x_3 & = & 18 \end{cases}$$

3. (a) Find the rank of the following matrix:

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$$

(b) Is this matrix invertible? If so, what is the inverse?

4. We know the following values of the linear map $T : \mathbb{R}^2 \to \mathbb{R}^4$:

$$T(\begin{bmatrix} 7 \\ 6 \end{bmatrix}) = \begin{bmatrix} 5 \\ 1 \\ -2 \\ 10 \end{bmatrix} \qquad T(\begin{bmatrix} 2 \\ 3 \end{bmatrix}) = \begin{bmatrix} 4 \\ 8 \\ 2 \\ 8 \end{bmatrix}$$

Find the matrix of this linear transformation. (Hint: Firstly compute $T(e_1)$ and $T(e_2)$.)

5. (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that rotates a vector in $\mathbb{R}^2$ through the angle $\frac{\pi}{4}$ clockwise. Let A be the matrix of this transformation. Compute AB when B is the following matrix:

$$\begin{bmatrix} 2 & 1 & 2 \\ 4 & 6 & 0 \end{bmatrix}$$

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(b) Suppose l is the line whose angle with the x-axis is equal to $\frac{\pi}{3}$. The linear transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ maps a vector to its reflection about the line l. Compute the matrix of this linear transformation.

(c) $U : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that rotates a vector though the angle $\frac{\pi}{2}$ counter-clockwise and then projects the resulting vector to the x-axis. Compute the matrix of this linear transformation.

6. For what values of λ, the following matrix is invertible? When it is invertible, find the inverse.

$$\begin{bmatrix} 2 & 9-\lambda \\ 8 & 4 \end{bmatrix}$$

7. Determine whether the following matrix is invertible. If it is, compute the inverse:

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}$$

8. A, B, and C are 3×3 matrices. Their inverses are given as follows:

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix}$$

Find the inverse of ABC.