Mat 211

Midterm 2 Spring 2016

Name: ______________________ **ID:** ______________________

Problem	1 (15 points)	2 (15 points)	3 (10 points)	4 (20 points)	5 (15 points)	6 (25 + 10 points)	Total (100 + 10 points)
Score							

Instructions:

(1) Fill in your name and Stony Brook ID number at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(4) Answer all questions in the space provided. If you need more room use the blank backs of the pages.

(5) Show your work; correct answers alone will receive only partial credit.

1. In each part, a basis $\mathfrak{B}$ of $\mathbb{R}^2$ is given (you don't need to show $\mathfrak{B}$ is a basis). Find he $\mathfrak{B}$-coordinate of the vector v.

   (a) $\mathfrak{B} = \{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}\}, \ v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

   (b) $\mathfrak{B} = \{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \end{bmatrix}\}, \ v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

2. Find bases for the image and the kernel of the linear transformation defined by the following matrix:

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 3 & 4 & -6 \\ 4 & 5 & -7 \end{bmatrix}$$

3. Determine whether each statement below is correct or not. You don't need to justify your answer; just give a "yes" or "no" answer.

   (a) Let $C(\mathbb{R},\mathbb{R})$ be the vector space that consists of continuous functions $f : \mathbb{R} \to \mathbb{R}$. Define W as:

   $$W = \{f \in C(\mathbb{R},\mathbb{R}) \mid f(20) = 16\}$$

   Is W a vector subspace of $C(\mathbb{R},\mathbb{R})$?

   (b) Let $W \subset \mathbb{R}^3$ be defined as:

   $$W = \{(x,y,z) \in \mathbb{R}^3 \mid x \leq y \leq 2z\}$$

   Is W a vector subspace of $\mathbb{R}^3$?

   (c) Let P_2 be the vector space of degree at most 2 polynomials and $\mathfrak{B}$ be the basis for P_2 that consists of 1, x, and x^2. Is:

   $$T : P_2 \to \mathbb{R}^3$$
   $$T(Q) = [Q]_{\mathfrak{B}}$$

   (d) Let l^∞ be the linear space of all infinite sequences of numbers where addition and scalar multiplication is defined as:

   $$(x_0, x_1, x_2, \dots) + (y_0, y_1, y_2, \dots) := (x_0 + y_0, x_1 + y_1, x_2 + y_2, \dots)$$

   $$k(x_0, x_1, x_2, \dots) := (kx_0, kx_1, kx_2, \dots)$$

   Consider the map $T : l^\infty \to l^\infty$:

   $$T(x_0, x_1, x_2, \dots) := (0, x_1, x_3, \dots)$$

   Is the vector $(0, 1, 2, 3, \dots)$ in the image of T?

   (e) Let T be defined as in the previous part. Is the vector $(0, 1, 2, 3, \dots)$ in the kernel of T?

4. In each part, is the given set of vectors a basis for $\mathbb{R}^3$? If not, is it possible to find a subset of the given vectors to form a basis? Justify your answer; if you think it is not possible explain why. If you think it is possible, find a subset which is a basis.

(a)

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}$$

(b)

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix} \quad v_3 = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

(c)

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -2 \\ -3 \\ -1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 6 \\ 11 \\ 5 \end{bmatrix} \quad v_4 = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$$

5. Let A be the following 2×2 matrix:

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 6 \end{bmatrix}$$

and the basis $\mathfrak{B} = \{v_1, v_2\}$ consists of the following vectors:

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

Find the matrix B of the linear transformation $T(\vec{x}) = A\vec{x}$ with respect to the basis $\mathfrak{B}$.

6. Different parts of this question are not necessarily related to each other. So if you can't do one part, don't give up on the other parts.

(a) Show that the following set determines a basis for $\mathbb{R}^{2\times 2}$: (Hint: You can use the fact that $\dim(\mathbb{R}^{2\times 2}) = 4$.)

$$\mathfrak{B} := \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$$

(b) Find the $\mathfrak{B}$-coordinate of the following matrix:

$$\begin{bmatrix} 1 & 3 \\ 9 & 5 \end{bmatrix}$$

(c) Show $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$, defined as follows, is a linear transformation.

$$T(M) = M \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} M$$

(d) Is T an isomorphism? Justify your answer.

(e) Find the $\mathfrak{B}$-matrix of the linear transformation T.

(f) (Bonus question) Find a basis for the kernel and the image of T.