Practice Exam for Midterm 1

1. Solve the linear systems using augmented matrices. State whether the solution is unique, there are no solutions or whether there are infinitely many. If the solution is unique give it. If there are infinitely many give the solution parametrically.

(a) $$\left\{\begin{array}{rcrcrcr} x_1 & & & - & 3x_3 & = & 8 \\ 2x_1 & + & 2x_2 & + & 9x_3 & = & 7 \\ & & x_2 & + & 5x_3 & = & -2 \end{array}\right.$$

(b) $$\left\{\begin{array}{rcrcrcr} 3x_1 & - & 4x_2 & + & 2x_3 & = & 0 \\ -9x_1 & + & 12x_2 & - & 6x_3 & = & 0 \\ -6x_1 & + & 8x_2 & - & 4x_3 & = & 0 \end{array}\right.$$

2. We know the following about the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$:

$$T(\begin{bmatrix} 2 \\ 3 \end{bmatrix}) = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad T(\begin{bmatrix} -1 \\ 4 \end{bmatrix}) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Determine the image of the following vectors with respect to the linear map T:

$$\begin{bmatrix} -1 \\ 15 \end{bmatrix}$$

3. Compute the product $A \cdot B$ of the following matrices:

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$$

4. (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that rotates a vector in $\mathbb{R}^2$ through the angle $\frac{\pi}{3}$ clockwise. Let A be the matrix of this transformation. Compute A.

(b) $S : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that reflects a vector about the y-axis. Let B be the matrix of this transformation. Compute B.

(c) $U : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that rotates a vector $\frac{\pi}{3}$ clockwise and then reflects the resulting vector about the y-axis. Let C be the matrix of this transformation. Compute C.

(d) Consider the matrix:

$$M = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 4 & 5 \end{bmatrix}$$

Compute the matrices AM, BM, and CM.

5. Determine whether the following matrix is invertible. If it is, compute the inverse:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

6. Find the rank of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 6 \\ 3 & 8 & 12 \end{bmatrix}$$

7. For what values of λ the following matrix is invertible?

$$\begin{bmatrix} \lambda - 1 & 2 \\ 3 & \lambda \end{bmatrix}$$

8. Suppose A and B are invertible linear transformations. Show that ABA^{-1} is also invertible.