Problem Set 4

1. Suppose $F : \mathbb{R}^2 \to \mathbb{R}^3$ is the function given by:

$$F\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4x^3y^2 \\ 2e^x + 3y \\ \frac{x}{y^2+1} \end{bmatrix}$$

(i) Show that F is differentiable at any point $\mathbf{c} = (c_1,c_2) \in \mathbb{R}^2$.

(ii) Find the Jacobian of F at the point $\mathbf{c} = (-1,2)$.

(iii) Find the derivative $D_{\mathbf{c}}F(\mathbf{v})$ where $\mathbf{c} = (-1,2)$ and $\mathbf{v} = (-2,1)$.

2. In each of the following problems, find $D_{\mathbf{c}}F(\mathbf{v})$ using the given data.

(i) $F : \mathbb{R}^2 \to \mathbb{R}^2$, $F\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = G\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) + \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $D_{\mathbf{c}}G(\mathbf{v}) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.

(ii) $F = \dfrac{G}{f}$ where $f : \mathbb{R}^2 \to \mathbb{R}$ and $G : \mathbb{R}^2 \to \mathbb{R}^2$, $f(\mathbf{c}) = -2$, $D_{\mathbf{c}}f(\mathbf{v}) = -1$, $D_{\mathbf{c}}G = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, $G(\mathbf{c}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.