

Problem Set 7

If you need some practice problems, you can work on problems 18-24 in Section 3.9 of the textbook. Section 3.10 has solutions to these problems. Going over your notes and working on these practice problems would help you to get a review of the materials relevant for this problem set.

1. Diagonalize the following quadratic forms. Then determine the rank and signature of each of them.

(i) $Q(x,y) = x^2 + 3xy + 3y^2$.

(ii) $Q(x,y,z) = xy + yz + zx$.

(iii) $Q(x,y,z) = 3x^2 + 3y^2 + 6xz - 2yz$.

2. The function $f(x,y) = x^3 - x^2 + y^2$ has two critical points. Find both, and classify each as a local maximum, local minimum, or saddle point.

3. Let $f(x,y) = x^3 + 3xy + y^3$. Find all critical points of f , and classify each as a local minimum, local maximum, or saddle point.

4. Let $f(x,y,z) = x^2 + y^2 + z^2 + xyz$.

(i) Show that $(0,0,0)$, $(-2, -2, -2)$, and $(-2,2,2)$ are critical points of f .

(ii) Classify each of the points in part (i) as a local minimum, local maximum, or saddle point.

5. Consider the general quadratic form $Q(x,y) = ax^2 + bxy + cy^2$, and assume $a > 0$ and $c > 0$. Find (in terms of a and c) a value b_0 such that Q has signature $(2,0)$ if $|b| < b_0$, and Q has signature $(1,1)$ if $|b| > b_0$. Prove your answer.