Problem Set 9

1. (i) Compute the volume of the following 3-dimensional solid using an appropriate linear change of varible.

$$\{(x,y,z) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\}.$$

(ii) Compute the area of the tringnle Δ in $\mathbb{R}^3$ which has vertices (0,0,0), (1,2,3) and $(1, -1, 0)$.

2. Let $\mathcal{S}$ be the half-circle $\{(x,y) \mid x^2 + y^2 < 1,\ x < \sqrt{3}y\}$. Find the integral

$$\int_{\mathcal{S}} x\, dx\, dy,$$

using a change of variables to polar coordinates.

3. (i) Describe the following region in cylindrical coordinates.

$$\mathcal{S} = \{(x,y,z) \in \mathbb{R}^3 \mid z \leq \sqrt{x^2+y^2}, x^2+y^2 < 5, x < 0, y > 0\}$$

(ii) Describe the following region in spherical coordinates.

$$\mathcal{S} = \{(x,y,z) \in \mathbb{R}^3 \mid z \geq \sqrt{x^2+y^2}, z < 5, x > 0, y > 0\}$$

(iii) Describe the following region in cylindrical coordinates.

$$\mathcal{S} = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 < 1, -\frac{1}{2} < z < \frac{1}{2}\}$$

4. Compute the integral

$$\int_M \sqrt{x^2 + y^2}$$

where $M \subset \mathbb{R}^3$ is the following 2-dimensional manifold

$$M = \{(x,y,z) \in \mathbb{R}^3 \mid 0 < z < 3\pi, x = r\cos(z), y = r\sin(z) \text{ with } 0 < r < 2\}.$$

5. Find the length the curve in $\mathbb{R}^3$ parametrized as

$$\gamma : (-\pi,\pi) \to \mathbb{R}^3$$

with

$$\gamma(t) = (t, \cos(2t), \sin(2t)).$$