Introduction to Calculus of Several Variables

Midterm 1 Spring 2020

Name: ______________________ **ID:** ______________________

Instructions:

(1) Fill in your name and Washington University ID at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. 1/2, not 2/4) to receive full credit.

(4) Read each questions carefully. Answer all questions in the space provided. If you need more room use the blank backs of the pages.

(5) Show your work; correct answers alone will receive only partial credit.

(6) This exam has 5 extra credit points.

Problem	1 (25 pts)	2 (25 pts)	3 (25 pts)	4 (30 pts)	Total (105 pts)
Score					

1. Find an orthonormal basis for the following subspace of $\mathbb{R}^4$:

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \in \mathbb{R}^4 \mid x + 2y - 6z + 2w = 0,\ y - 2z + 2w = 0 \right\}$$

2. In each part, a subspace of $\mathbb{R}^2$ is given. Firstly **sketch** the given region and then **determine its boundary**.

(i) $A = \{(x, y) \in \mathbb{R}^2 \mid -1 < x < 1,\ 1 \leq y \leq 1\}$.

(ii) $B = \{(x, y) \in \mathbb{R}^2 \mid y = x^2\}$.

3. Determine whether the given functions $f : \mathbb{R}^2 \to \mathbb{R}$ are continuous at the point (0,0)

(i) $$f(x,y) = \begin{cases} \frac{\sin(x)\cos(y)}{x} & x \neq 0 \\ \cos(y) & x = 0 \end{cases}.$$

(ii) $$f(x,y) = \begin{cases} \frac{x^3 y}{x^4+y^4} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$$

4. (i) Show that $F : \mathbb{R}^2 \to \mathbb{R}^2$ is differentiable everywhere.

$$F\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 - xy \\ \cos(x) + 2y \end{bmatrix}.$$

(State clearly what criterion you use to show this claim.)

(ii) Find the Jacobian of the map F from part (i)

(iii) Suppose $\mathbf{c} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Suppose the function $G : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies:

$$G(\mathbf{c}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad D_{\mathbf{c}}G(\mathbf{v}) = \begin{bmatrix} 2 \\ 0 \end{bmatrix},$$

and the function $f : \mathbb{R}^2 \to \mathbb{R}$ satisfies:

$$f(\mathbf{c}) = 3, \qquad D_{\mathbf{c}}f(\mathbf{v}) = -1.$$

Let $H : \mathbb{R}^2 \to \mathbb{R}^2$ be the function $f \cdot G$. Determine $D_{\mathbf{c}}H(\mathbf{v})$. (State clearly what property of derivatives you use. Notice that the functions here **do not** have anything to do with the functions in the first two parts.)