Set Theory

Let X_1, X_2, X_3, X_4 and X_5 be the subsets of the set U of all months which are defined as follows.

$X_1 = \{\text{February, March, June, September}\}$.

$X_2 = \{x \mid x \text{ is a month with exactly 30 days}\}$.

$X_3 = \{x \mid x \text{ is a month which overlaps with summer}\}$.

$X_4 = \{\text{August, December, March}\}$.

$X_5 = \{\text{January, February, May, July, October}\}$.

1. Find the following subsets of U:

   (a) $X_1 \cup X_2$;

   (b) $X_3 \cup X_4$;

   (c) $\displaystyle\bigcup_{i=1}^{4} X_i$;

   (d) X_5^c, the complement of X_5;

   (e) $\displaystyle\bigcap_{1 \le i \le 3} X_i$;

(f) $X_1 \times X_2$;

(g) $X_1 \setminus X_2$;

2. Determine whether the following statements are correct:

(a) The intersection of the sets X_1, X_2, X_3 and X_4 is the empty set.

(b) X_1 and $X_3 \cap X_4$ are disjoint;

(c) X_1, X_2 and $X_3 \cap X_4$ are mutually disjoint;

(d) X_2, X_4 and X_5 gives a partition of U.